Imperial College London

Instituto Tecnológico de Aeronáutica - ITA - Brasil

Under-resolved DNS of non-trivial turbulent boundary layers via spectral/hp continuous Galerkin methods

Rodrigo C. Moura, Joaquim Peiró and Spencer Sherwin

Nektar++ Workshop
Exeter, UK - June 10 / 2019

Introduction & motivation

- (U)RANS is less reliable for “non-standard” flows
- Wall-modeled LES still unreliable for certain flows [*]
- Wall-resolved LES too expensive at high Reynolds
- Wall-resolved implicit LES / under-resolved DNS via spectral/hp methods are promising strategies (CG, DG, etc)
- They seem to offer reliable results for highly complex flows at a reduced cost (being affordable at moderate Reynolds)

J. Larsson, S. Kawai, J. Bodart, and I. Bermejo-Moreno. Large eddy simulation with modeled wall-stress: recent progress and future directions. *Mechanical Engineering Reviews*, 3(1):15, 2016.

Introduction & motivation

- (U)RANS is less reliable for "non-standard" flows
- Wall-modeled LES still unreliable for certain flows [*]
- Wall-resolved LES too expensive at high Reynolds
- Wall-resolved implicit LES / under-resolved DNS via spectral/hp methods are promising strategies (CG, DG, etc)
- They seem to offer reliable results for highly complex flows at a reduced cost (being affordable at moderate Reynolds)

Imperial College London

Introduction & motivation

- (U)RANS is less reliable for “non-standard” flows
- Wall-modeled LES still unreliable for certain flows [*]
- Wall-resolved LES too expensive at high Reynolds
- Wall-resolved <u>implicit LES / under-resolved DNS</u> via spectral/hp methods are promising strategies (CG, DG, etc)
- They seem to offer reliable results for highly complex flows at a reduced cost (being affordable at moderate Reynolds)

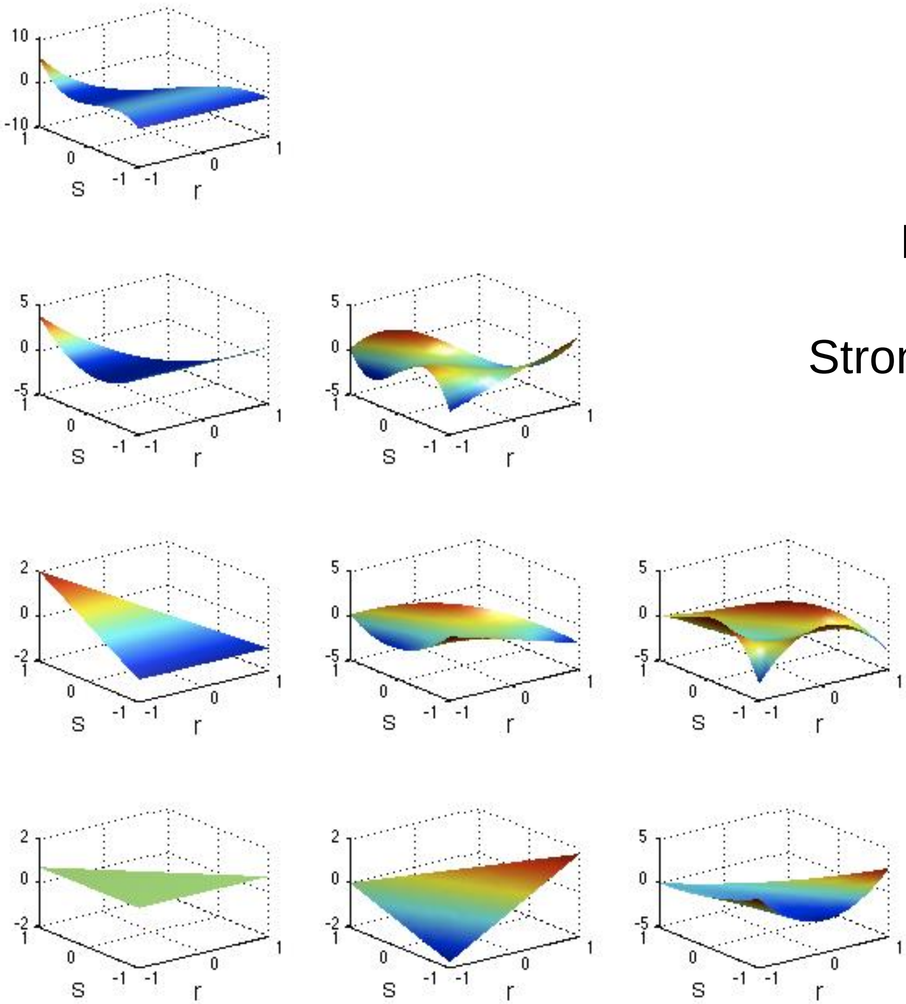
Presentation outline

- What are spectral element methods (SEM)?
- What is the SEM-based iLES / uDNS approach?
- How to understand its properties and capabilities?
- For DG, dispersion-diffusion eigenanalysis has proved insightful
- Ongoing eigenanalysis of CG with stabilisation based on SVV (and others)

Presentation outline

- What are spectral element methods (SEM)?
- What is the SEM-based iLES / uDNS approach?
- How to understand its properties and capabilities?
- For DG, dispersion-diffusion eigenanalysis has proved insightful
- Ongoing eigenanalysis of CG with stabilisation based on SVV (and others)
- Current work focuses on CG's performance for a specific test case
  (non-trivial turbulent BL case)

Spectral element methods

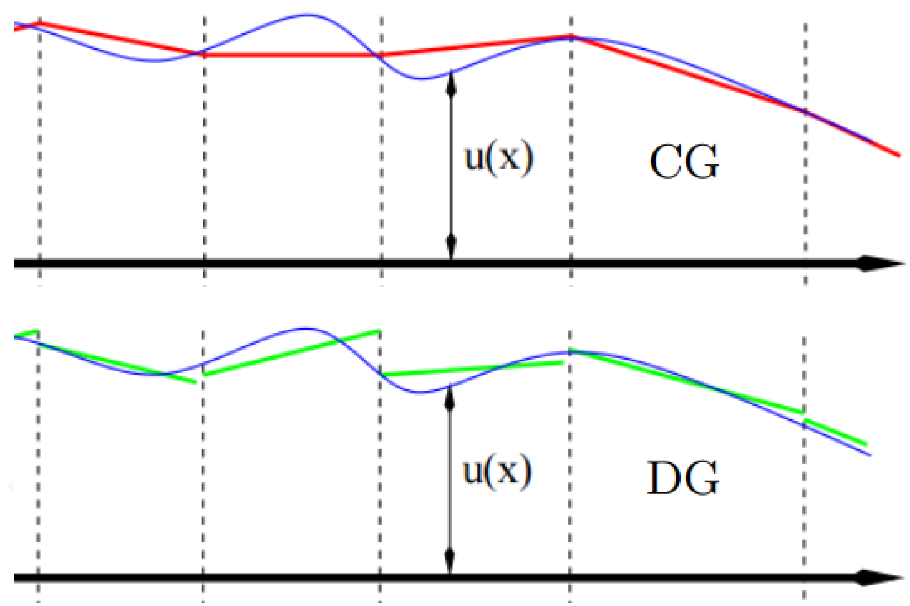


Extensions of FE / FV via polynomial basis

Kind of spectral method within the element

Natural high-order with geometrical flexibility

Strong / weak inter-element continuity (CG / DG)



Stabilisation for iLES / uDNS

Stabilization via SVV or upwind (Riemann) fluxes

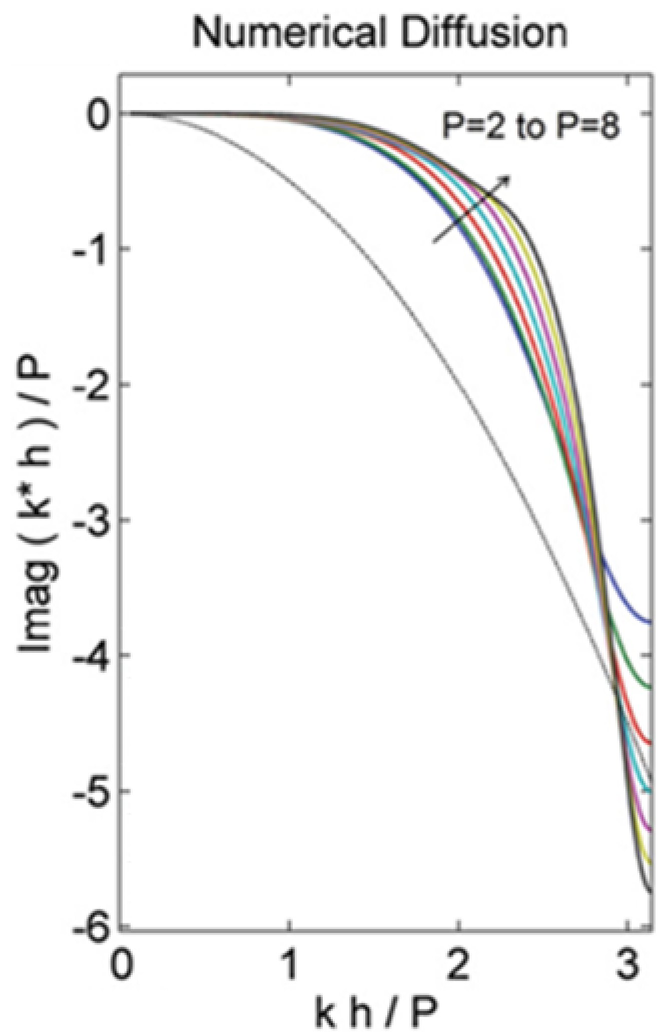
Hyperviscous-like dissipation at higher orders

Improved resolution power per DOF employed

Rationale for implicit LES / under-resolved DNS

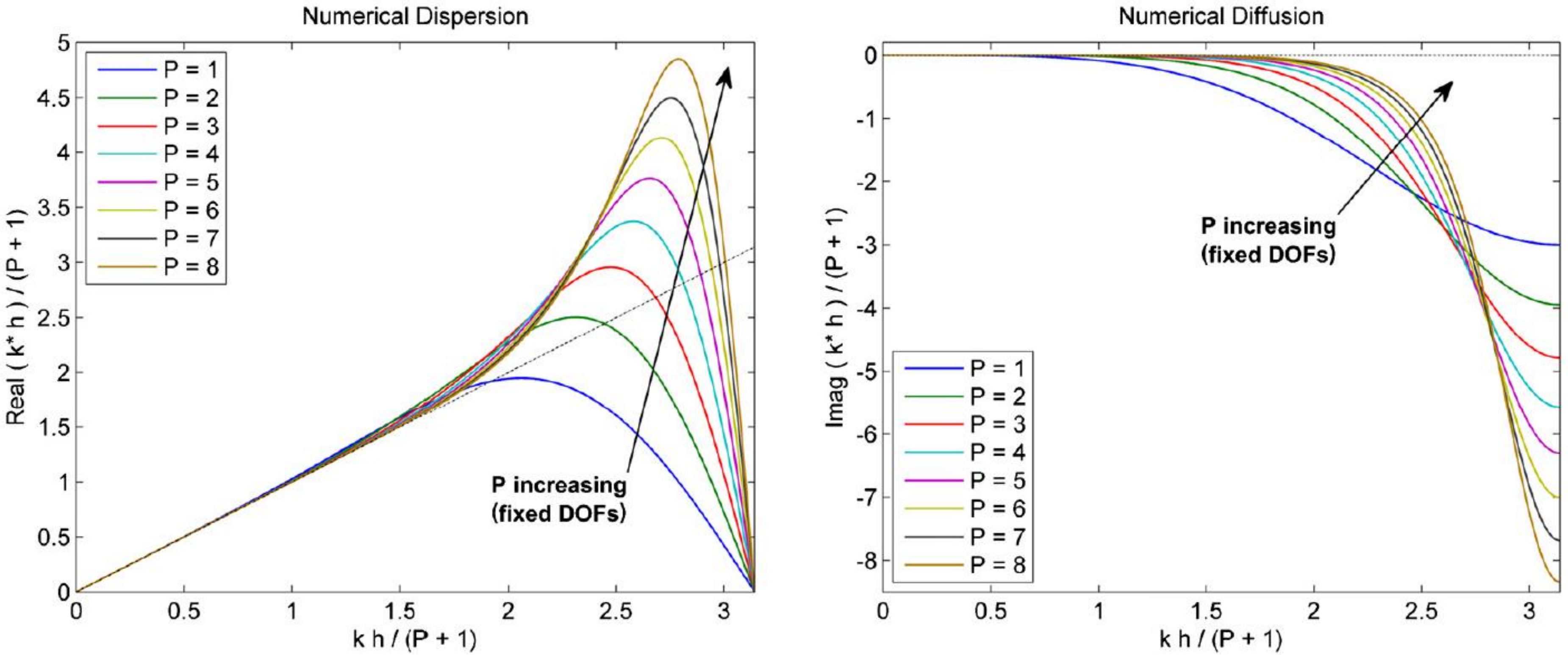
$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = \mu \frac{\partial}{\partial x}\left(\mathcal{Q} \star \frac{\partial u}{\partial x}\right) \approx$$

$$\approx -\mu \sum_k k^2\, \hat{\mathcal{Q}}_k\, \hat{u}_k \exp(ikx)$$



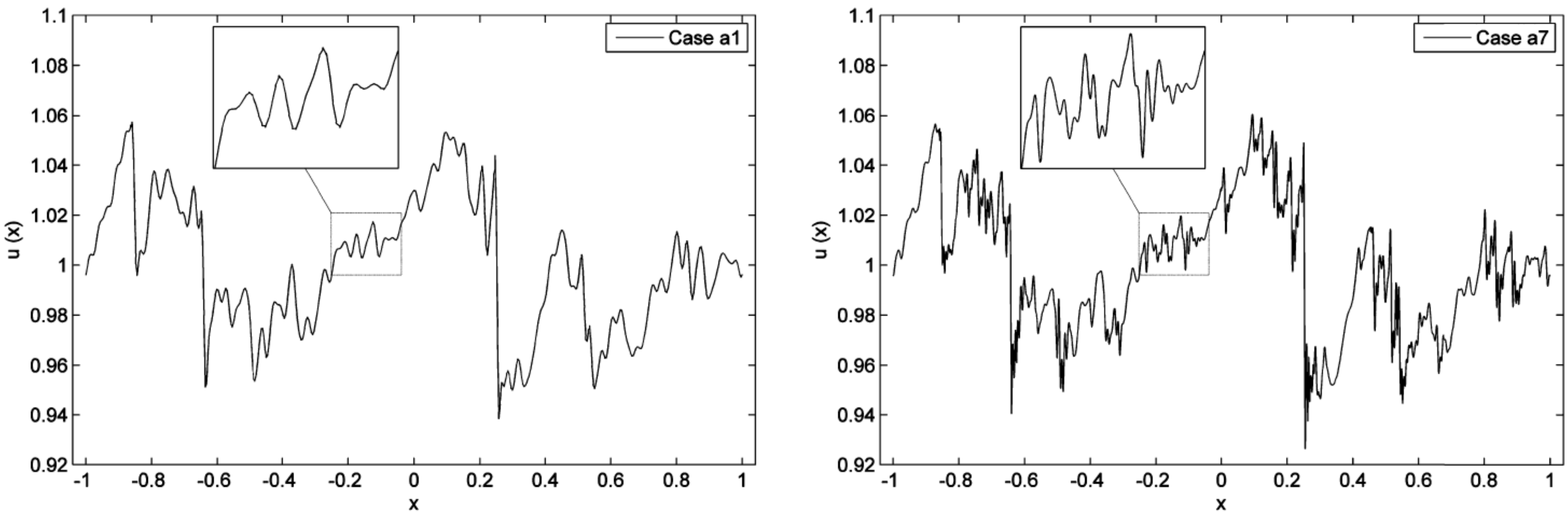
Imperial College London

Eigenanalysis for DG – linear advection in 1D



Tests with Burgers turbulence (1D)

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial x} = \frac{A_F}{\sqrt{\Delta t}} \sum_{N \in \mathbb{N}_F} \frac{\sigma_N(t)}{\sqrt{|N|}} \exp\left(i \frac{2\pi N}{L} x\right)$$

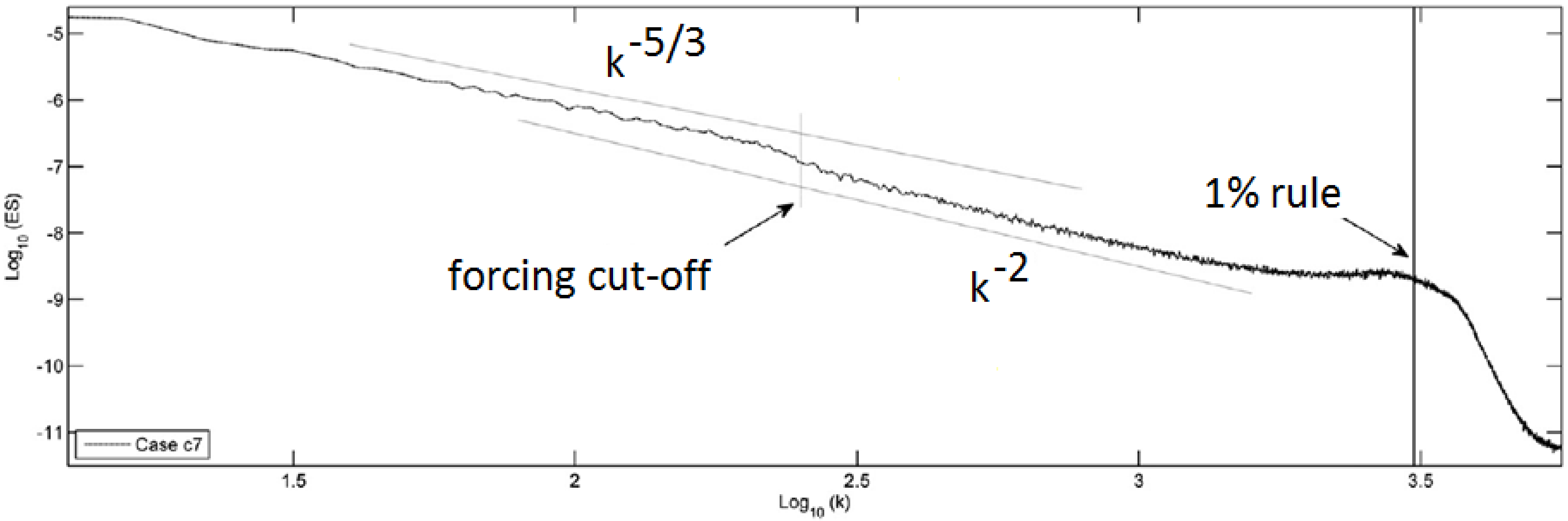


$p = 1, n_{el} = 2048$

$p = 7, n_{el} = 512$

Tests with Burgers turbulence (1D)

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial x} = \frac{A_F}{\sqrt{\Delta t}} \sum_{N \in \mathbb{N}_F} \frac{\sigma_N(t)}{\sqrt{|N|}} \exp\left(i\,\frac{2\pi N}{L} x\right)$$



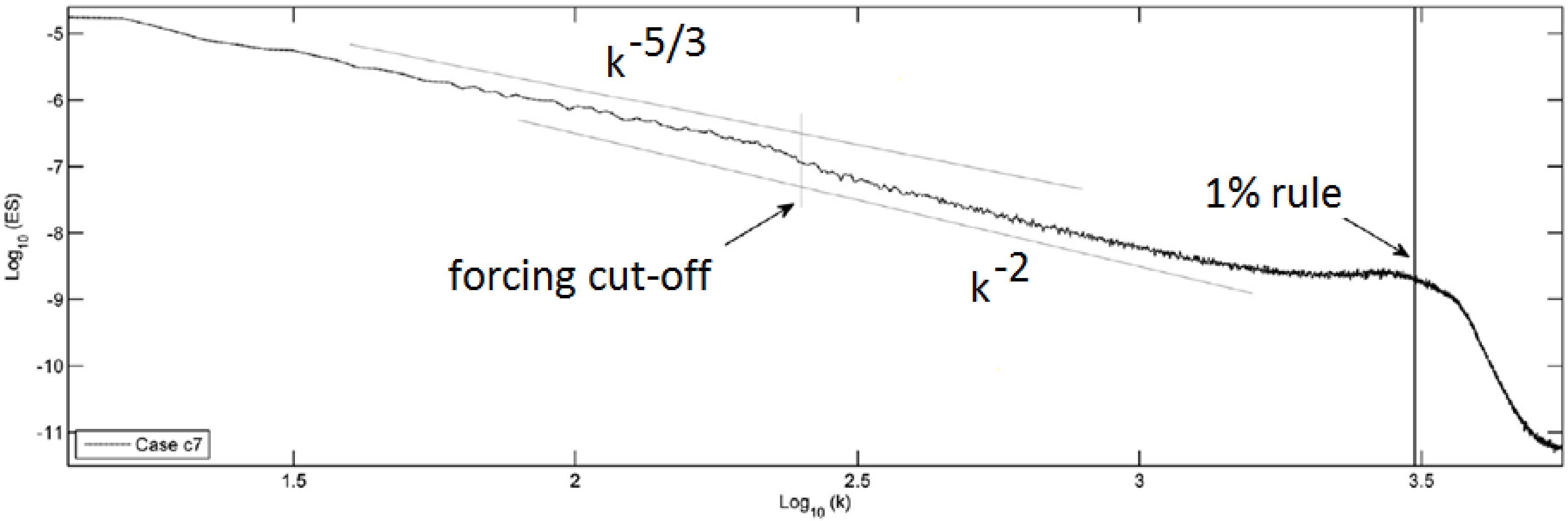
$$p = 7, n_{el} = 512$$

Imperial College London

R. C. Moura, S. J. Sherwin, and J. Peiró. Linear dispersion-diffusion analysis and its application to under-resolved turbulence simulations using discontinuous Galerkin spectral/*hp* methods. *Journal of Computational Physics*, 298:695–710, 2015.

Tests with Burgers turbulence (1D)

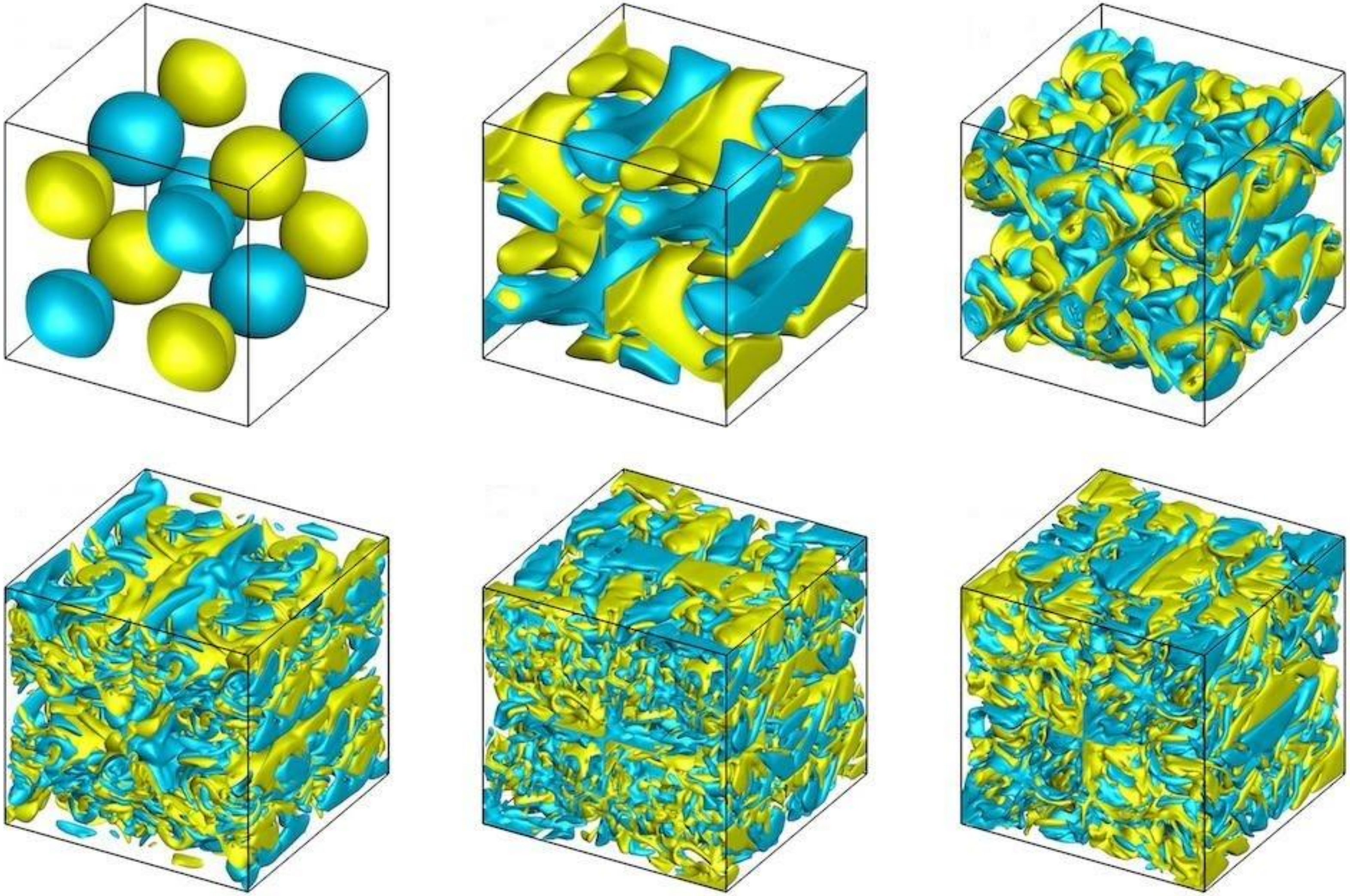
$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial x} = \frac{A_F}{\sqrt{\Delta t}} \sum_{N \in \mathbb{N}_F} \frac{\sigma_N(t)}{\sqrt{|N|}} \exp\left(i \frac{2\pi N}{L} x\right)$$



$$p = 7, n_{el} = 512$$

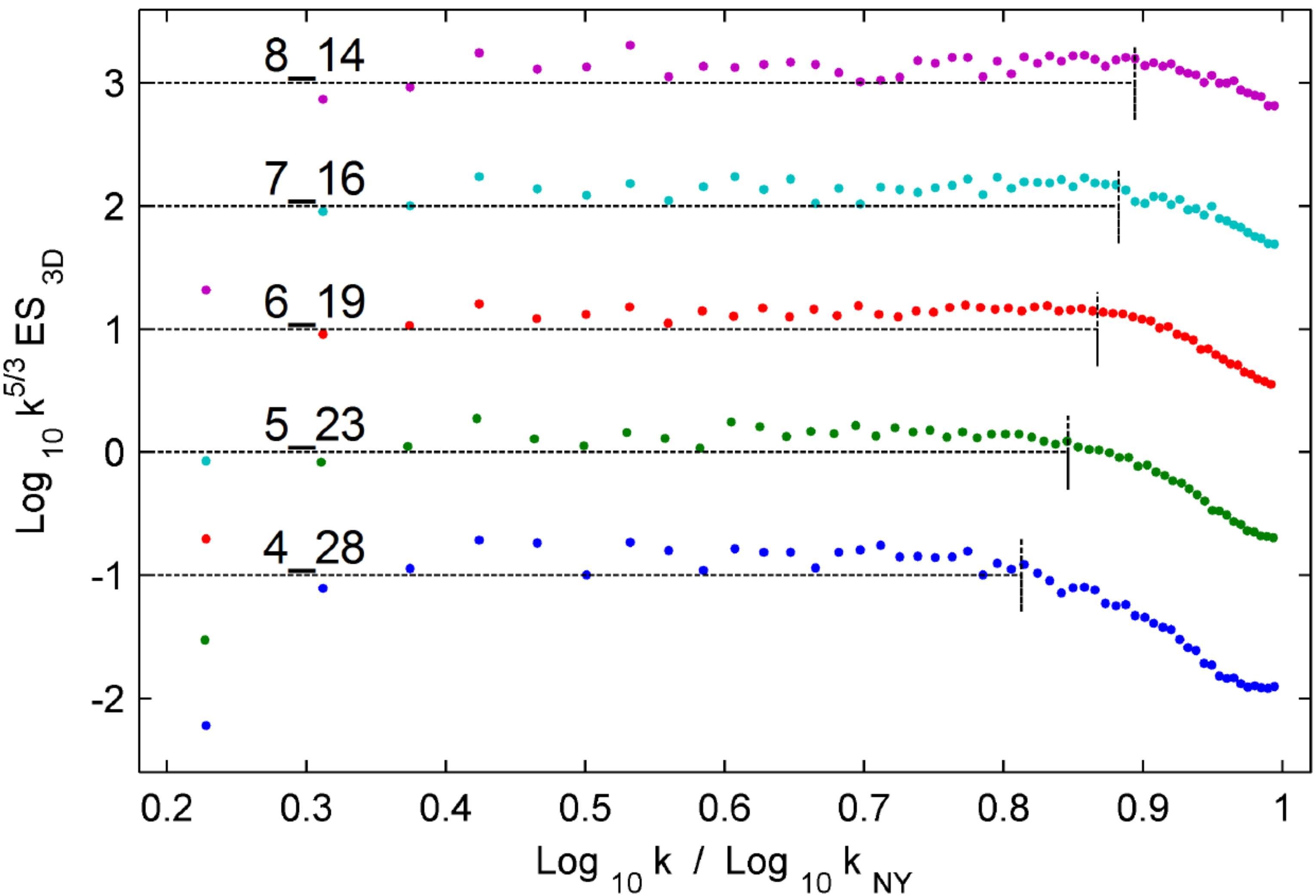
Imperial College London

Tests with the inviscid TGV flow



Imperial College London

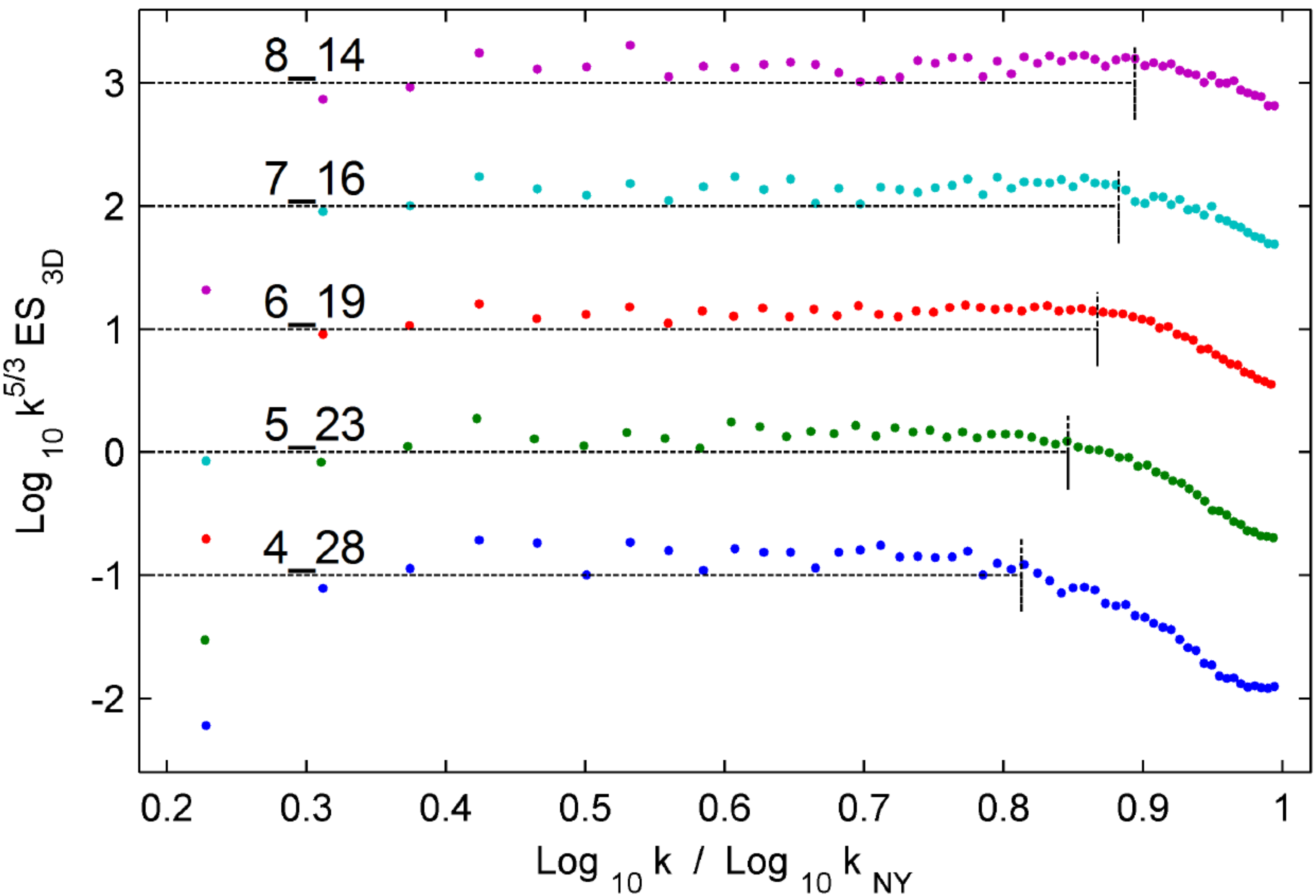
3D energy spectra – adapted 1% rule (3D version)



Imperial College London

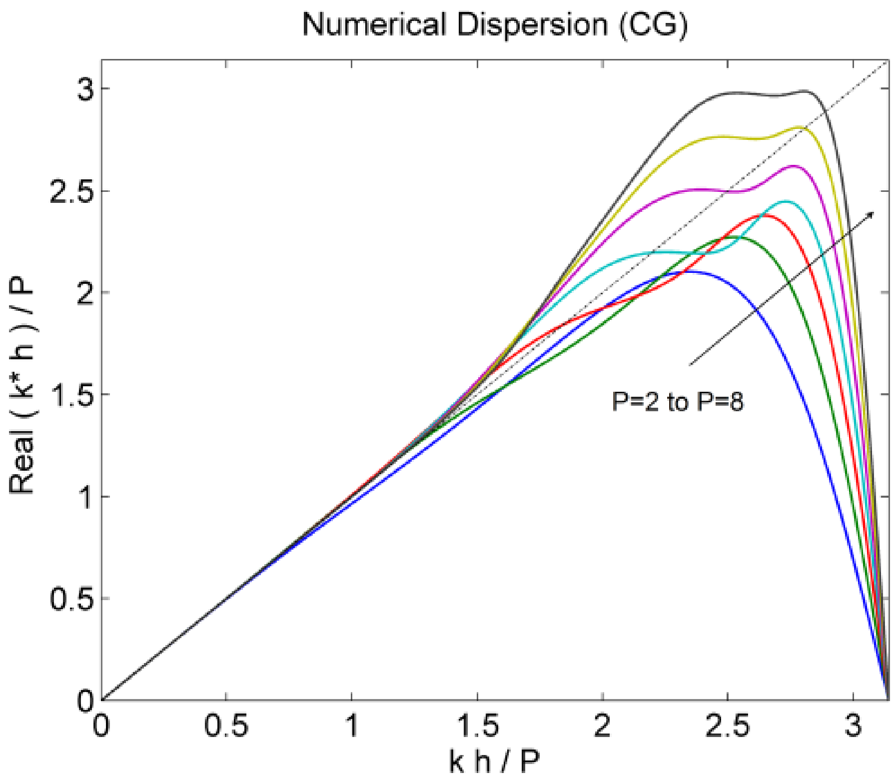
R. C. Moura, G. Mengaldo, J. Peiró, and S. J. Sherwin. On the eddy-resolving capability of high-order discontinuous Galerkin approaches to implicit LES / under-resolved DNS of Euler turbulence. *Journal of Computational Physics*, 330:615–623, 2017.

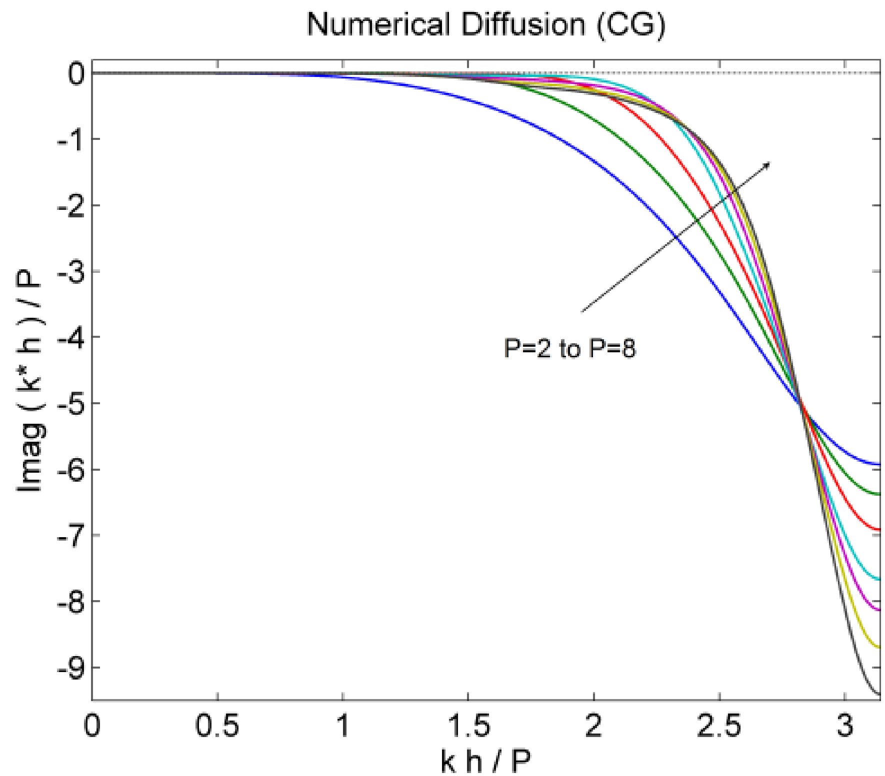
3D energy spectra – adapted 1% rule (3D version)



Imperial College London

Eigenanalysis for CG – linear advection plus SVV

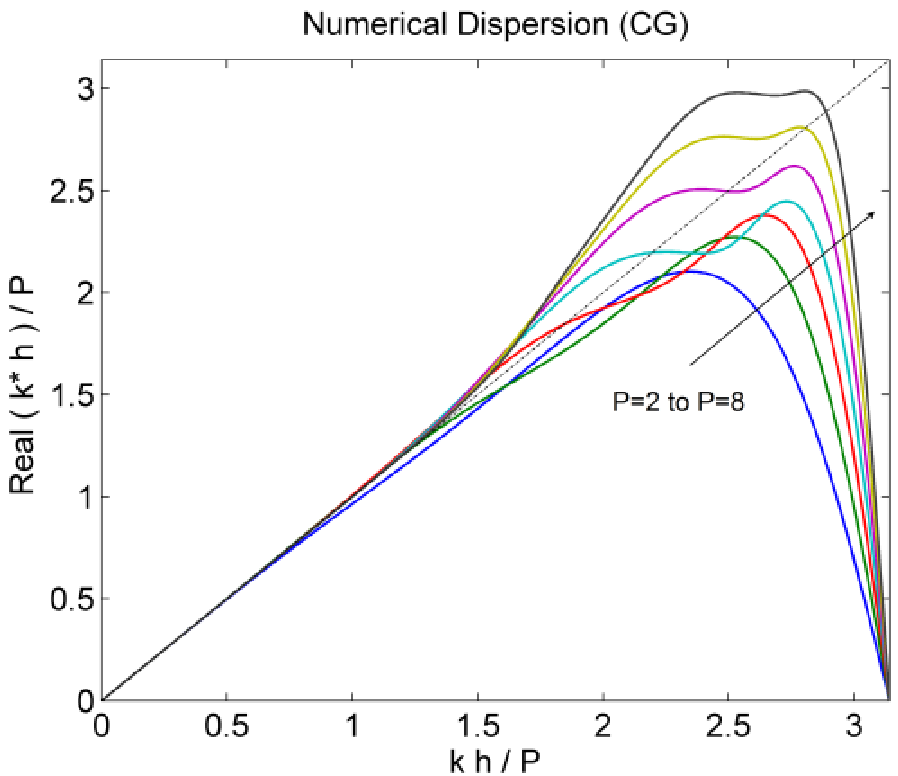


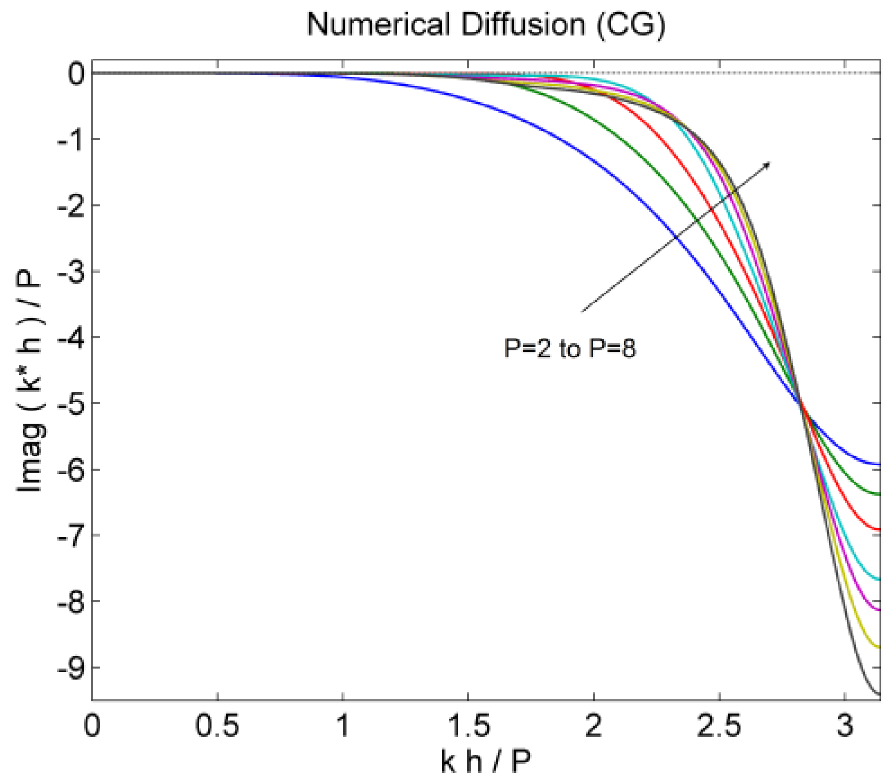


Imperial College London

R. C. Moura, S. J. Sherwin, and J. Peiró. Eigensolution analysis of spectral/*hp* continuous Galerkin approximations to advection-diffusion problems: insights into spectral vanishing viscosity. *Journal of Computational Physics*, 307:401–422, 2016.

Eigenanalysis for CG – linear advection plus SVV





Imperial College London

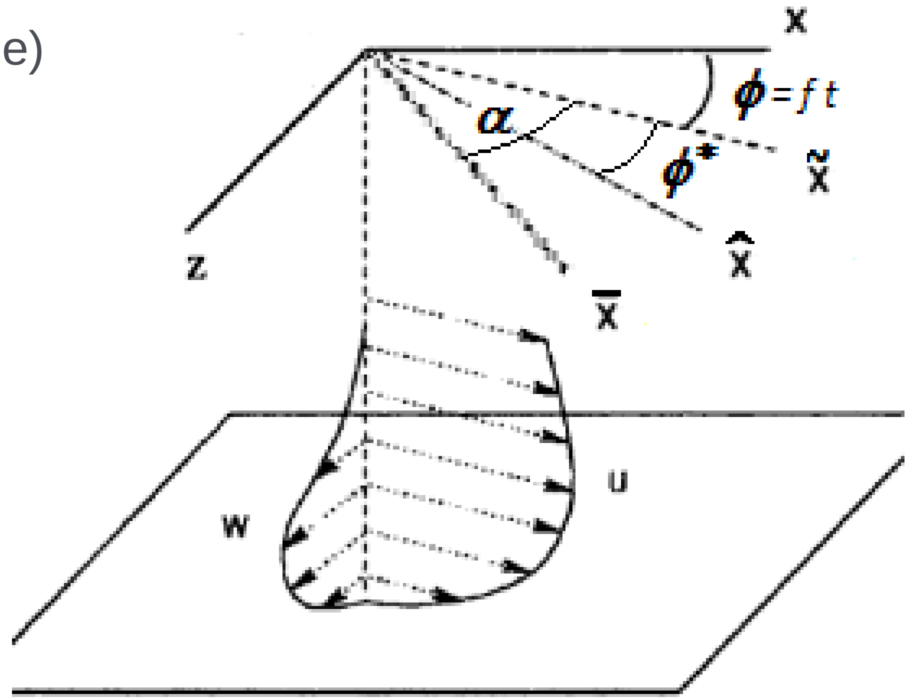
Presentation outline

- What are spectral element methods (SEM)?
- What is the SEM-based iLES / uDNS approach?
- How to understand its properties and capabilities?
- For DG, dispersion-diffusion eigenanalysis has proved insightful
- Ongoing eigenanalysis of CG with SVV-based stabilisation
- Current work focuses on CG's performance for a specific test case
  (non-trivial turbulent BL case)

Test case description

- Rotating turbulent BL case
  proposed by Spalart [*]
- Admits an asymptotic solution
  (stat. steady in the rotating frame)
- Rich physics, enhanced unsteadiness
  and stronger anisotropy
- Misalignment between mean-flow
  shear and Reynolds stresses
- Requires sophisticated modeling
  and relatively fine grids

$$u(x,z,t) = V_o \cos\phi$$
$$w(x,z,t) = V_o \sin\phi$$
$$\phi = ft$$



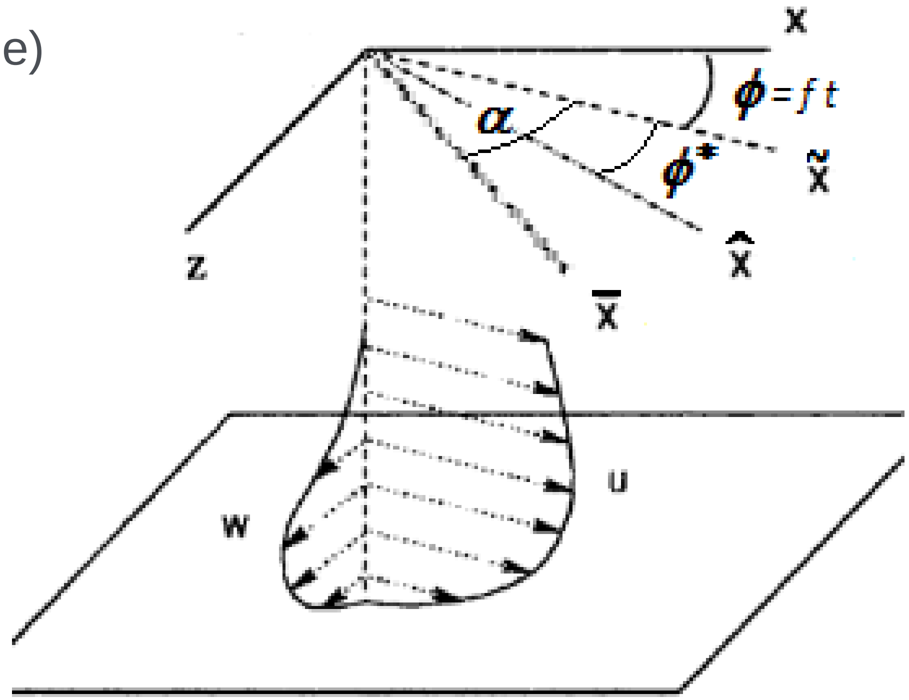
Imperial College London

Philippe R Spalart. Theoretical and numerical study of a three-dimensional turbulent boundary layer. *Journal of Fluid Mechanics*, 205:319–340, 1989.

Test case description

- Rotating turbulent BL case
  proposed by Spalart [*]
- Admits an asymptotic solution
  (stat. steady in the rotating frame)
- Rich physics, enhanced unsteadiness
  and stronger anisotropy
- Misalignment between mean-flow
  shear and Reynolds stresses
- Requires sophisticated modeling
  and relatively fine grids

$$u(x,z,t) = V_o \cos\phi$$
$$w(x,z,t) = V_o \sin\phi$$
$$\phi = ft$$



Imperial College London

Relevant quantities and available DNSs

- Known laminar solution gives $\delta_L = (2\, \nu / f\,)^{1/2}$
- Several Reynolds can be defined: R_L , R_T … $Re = \delta_T V_O / \nu$
- Summary of relevant data from available DNS results:

R_ℓ	R_T	Re	δ_T/δ_ℓ	V_*/V_o	δ_*/δ_ℓ	α (°)	ϕ^* (°)
500	466	7632	15.3	0.0610	0.0328	26.24	19.88
620	653	11203	18.1	0.0583	0.0277	23.20	18.62
767	914	16397	21.4	0.0561	0.0232	20.91	17.70

Relevant quantities and available DNSs

- Known laminar solution gives $\delta_L = (2 \nu / f)^{1/2}$
- Several Reynolds can be defined: R_L , R_T … $Re = \delta_T V_O / \nu$
- Summary of relevant data from available DNS results:

R_ℓ	R_τ	Re	δ_τ/δ_ℓ	V_*/V_o	δ_*/δ_ℓ	α (°)	ϕ^* (°)
500	466	7632	15.3	0.0610	0.0328	26.24	19.88
620	653	11203	18.1	0.0583	0.0277	23.20	18.62
767	914	16397	21.4	0.0561	0.0232	20.91	17.70

Available solutions at $R_L = 767$

- In previous LESs, the Mixed Lagrangian (ML) model gave best results [*]

Approach	Model	Discretisation	L_x , L_z	L_y	Mesh points	Δx^+, Δz^+
DNS [40]	—	Spectral method [49]	$2\,\delta_t$	?	$256^2 \cdot 80$	7
LES [41]	ML	2^{nd} order central FD	$1.6\,\delta_t$	$1.6\,\delta_t$	65^3	23
LES [42]	DA	2^{nd} order central FD	$1.6\,\delta_t$	$1.6\,\delta_t$	66^3	22

(first node at $y^+ = 1$ and 10 equispaced nodes within $y^+ < 10$, then a constant geometric stretching along the remainder of the BL thickness)

X. Wu and K. D. Squires. Large eddy simulation of an equilibrium three-dimensional turbulent boundary layer. *AIAA Journal*, 35(1):67–74, 1997.

Available solutions at $R_L = 767$

- In previous LESs, the Mixed Lagrangian (ML) model gave best results [*]

Approach	Model	Discretisation	L_x , L_z	L_y	Mesh points	Δx^+, Δz^+
DNS [40]	—	Spectral method [49]	$2\,\delta_t$	?	$256^2 \cdot 80$	7
LES [41]	ML	2^{nd} order central FD	$1.6\,\delta_t$	$1.6\,\delta_t$	65^3	23
LES [42]	DA	2^{nd} order central FD	$1.6\,\delta_t$	$1.6\,\delta_t$	66^3	22

(first node at $y^+ = 1$ and 10 equispaced nodes within $y^+ < 10$, then a constant geometric stretching along the remainder of the BL thickness)

X. Wu and K. D. Squires. Large eddy simulation of an equilibrium three-dimensional turbulent boundary layer. *AIAA Journal*, 35(1):67–74, 1997.

Available solutions at $R_L = 767$

- In previous LESs, the Mixed Lagrangian (ML) model gave best results [*]

Approach	Model	Discretisation	L_x , L_z	L_y	Mesh points	Δx^+, Δz^+
DNS [40]	—	Spectral method [49]	$2\,\delta_t$	?	$256^2 \cdot 80$	7
LES [41]	ML	2^{nd} order central FD	$1.6\,\delta_t$	$1.6\,\delta_t$	65^3	23
LES [42]	DA	2^{nd} order central FD	$1.6\,\delta_t$	$1.6\,\delta_t$	66^3	22

(first node at $y^+ = 1$ and 10 equispaced nodes within $y^+ < 10$, then a constant geometric stretching along the remainder of the BL thickness)

- Discretisations employed in the present work:
  - Run **32p1**: 8 equispaced P = 1 elements within $y^+ < 10$ and the remaining 24 elements stretched until $y = 1.6\,\delta_T$ … (total 32^3 dofs)

X. Wu and K. D. Squires. Large eddy simulation of an equilibrium three-dimensional turbulent boundary layer. *AIAA Journal*, 35(1):67–74, 1997.

Available solutions at $R_L = 767$

- In previous LESs, the Mixed Lagrangian (ML) model gave best results [*]

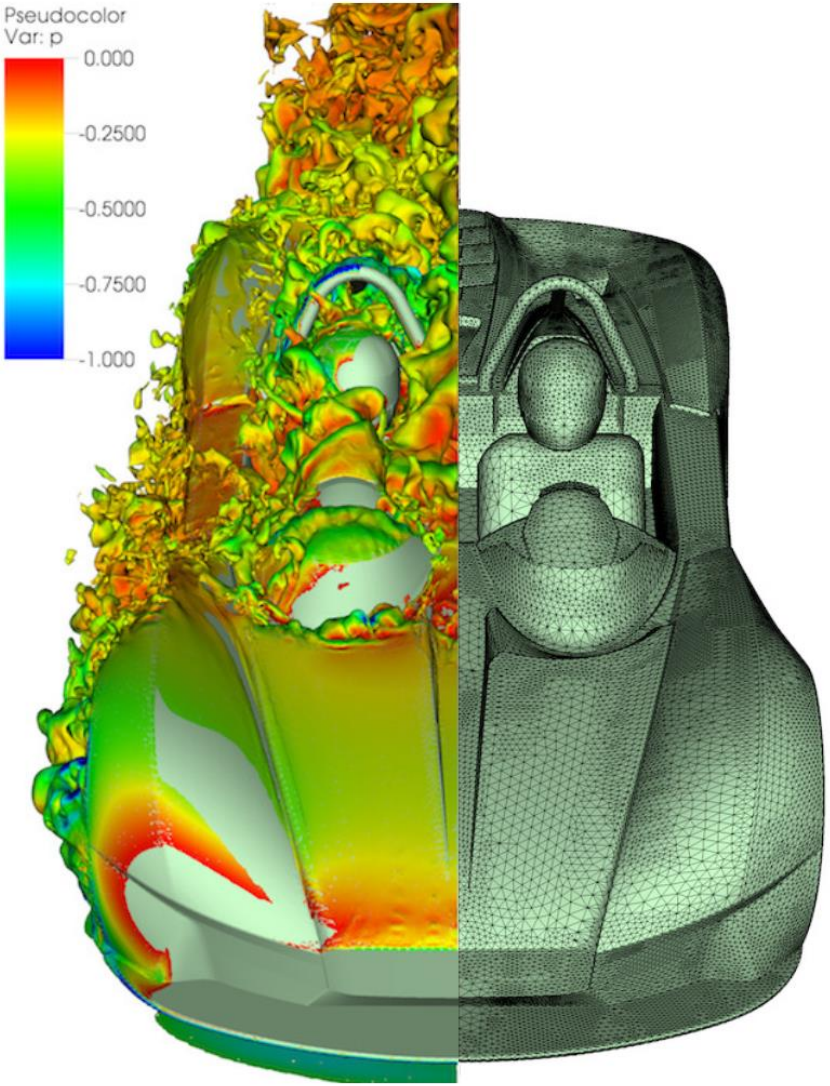
Approach	Model	Discretisation	L_x , L_z	L_y	Mesh points	Δx^+, Δz^+
DNS [40]	—	Spectral method [49]	$2\,\delta_t$	?	$256^2 \cdot 80$	7
LES [41]	ML	2^{nd} order central FD	$1.6\,\delta_t$	$1.6\,\delta_t$	65^3	23
LES [42]	DA	2^{nd} order central FD	$1.6\,\delta_t$	$1.6\,\delta_t$	66^3	22

(first node at $y^+ = 1$ and 10 equispaced nodes within $y^+ < 10$, then a constant geometric stretching along the remainder of the BL thickness)

- Discretisations employed in the present work:
  - Run **32p1**: 8 equispaced P = 1 elements within $y^+ < 10$ and the remaining 24 elements stretched until $y = 1.6\ \delta_T$ … (total 32^3 dofs)
  - Run **8p4**: 2 equispaced P = 4 elements within $y^+ < 10$ and the remaining 6 elements stretched until $y = 1.6\ \delta_T$ … (total 32^3 dofs)

Numerical set-up

- Nektar++ incompressible solver [*]
- Spectral/hp CG with P=1 and P=4
- Using polynomial dealiasing, but no SVV
- Perturbed laminar solution as IC
- Transient takes ~ 10 cicles
- Statistics gathered over ~ 1 cicle

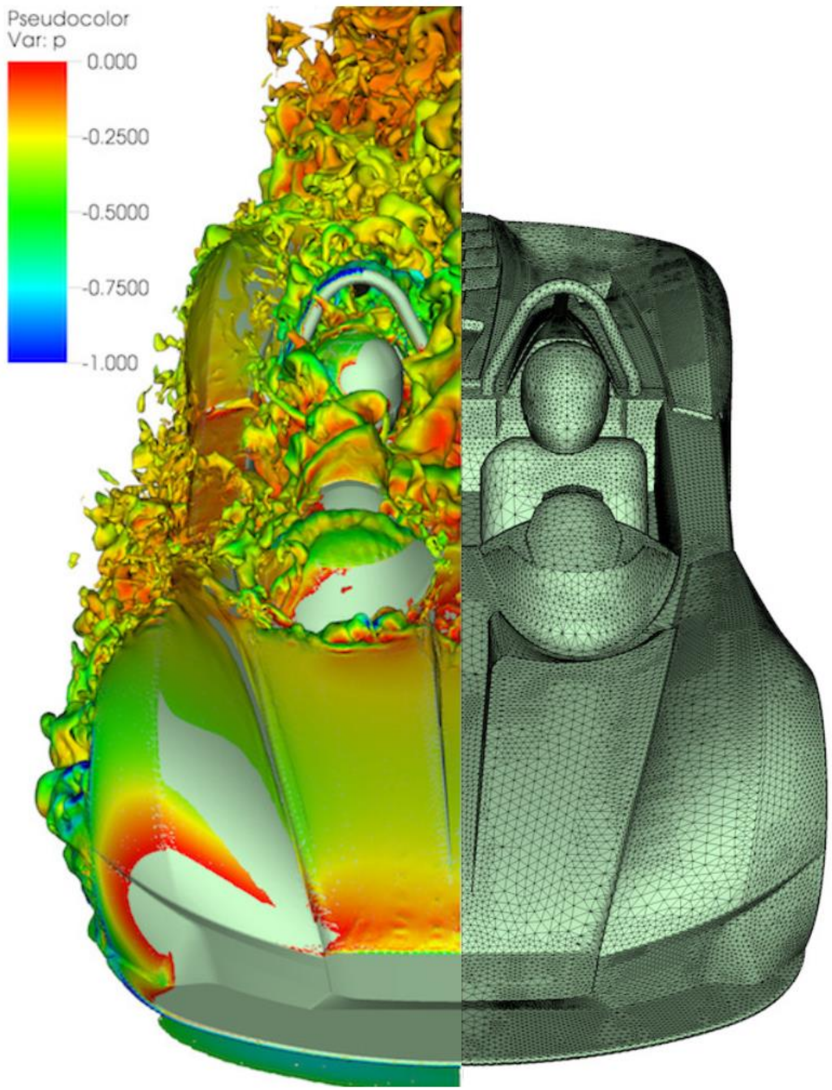


Imperial College London

C. D. Cantwell, D. Moxey, A. Comerford, A. Bolis, G. Rocco, G. Mengaldo, D. De Grazia, S. Yakovlev, J-E. Lombard, [redacted] and S. J. Sherwin. Nektar++: An open-source spectral/*hp* element framework. *Computer Physics Communications*, 192:205–219, 2015.

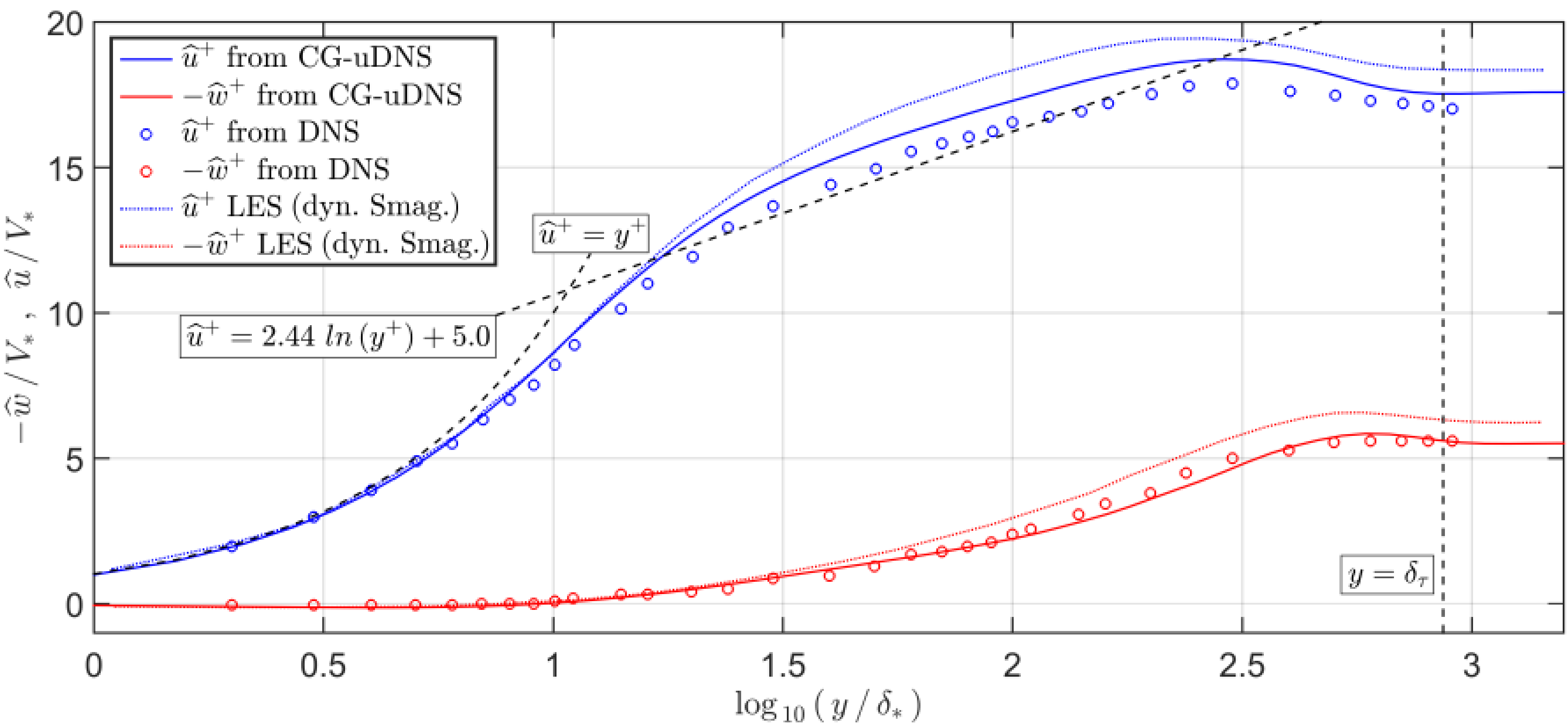
Numerical set-up

- Nektar++ incompressible solver [*]
- Spectral/hp CG with P=1 and P=4
- Using polynomial dealiasing, but no SVV
- Perturbed laminar solution as IC
- Transient takes ~ 10 cicles
- Statistics gathered over ~ 1 cicle



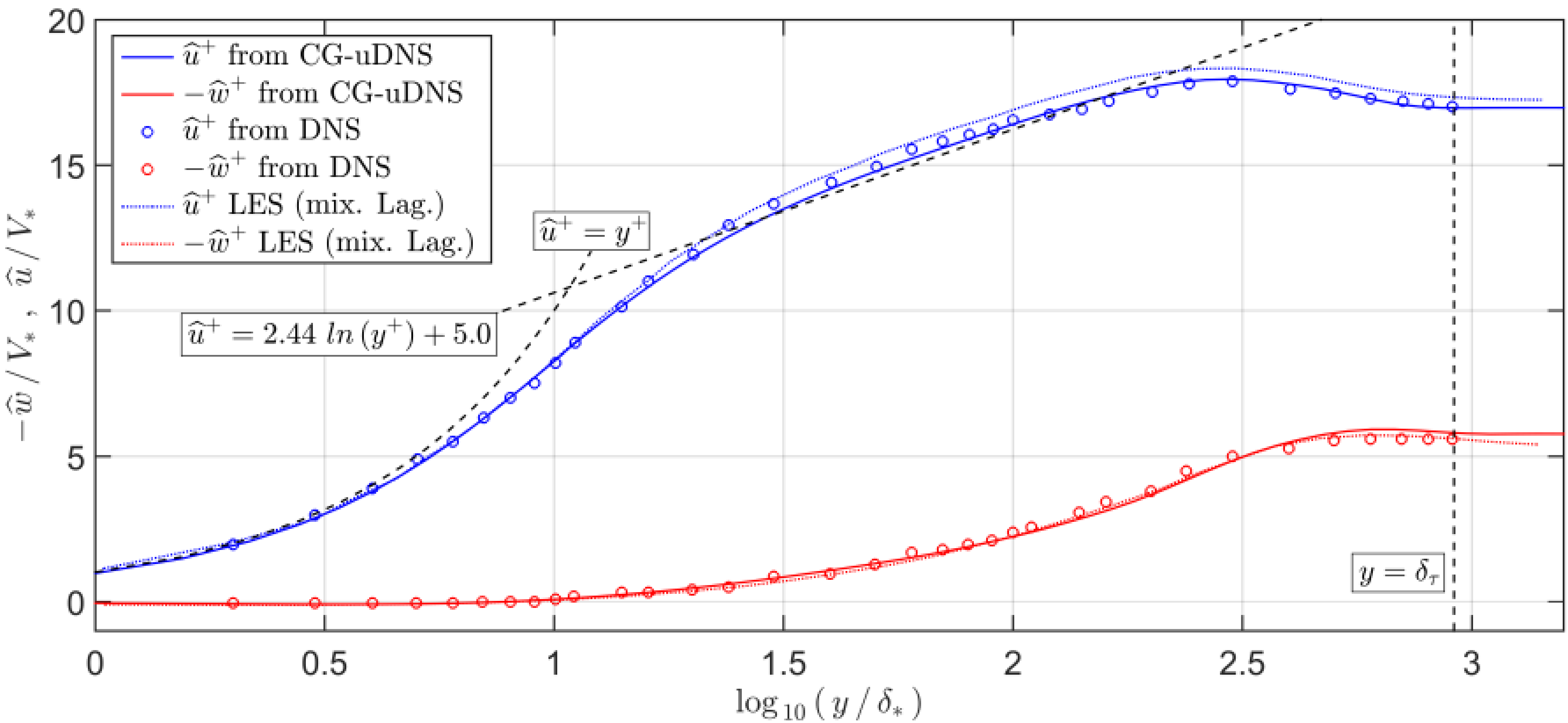
Results of run 32p1

CG-uDNS (p=1) on 32^3 dofs VS. dynamic Smagorinsky (FD-based) on 64^3 dofs



Results of run 8p4

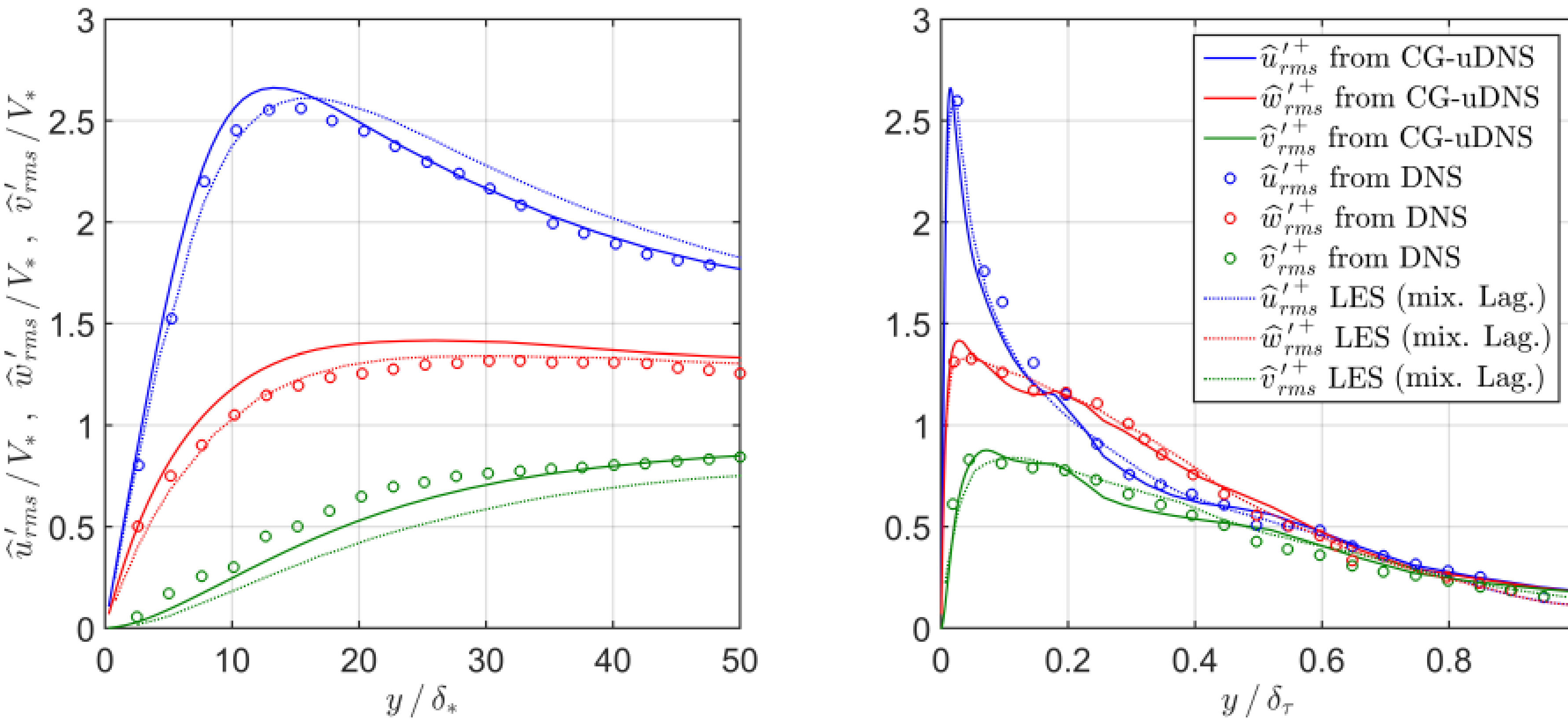
CG-uDNS (p=4) on 32^3 dofs VS. mixed Lagrangian model (FD-based) on 64^3 dofs



Imperial College London

Results of run 8p4

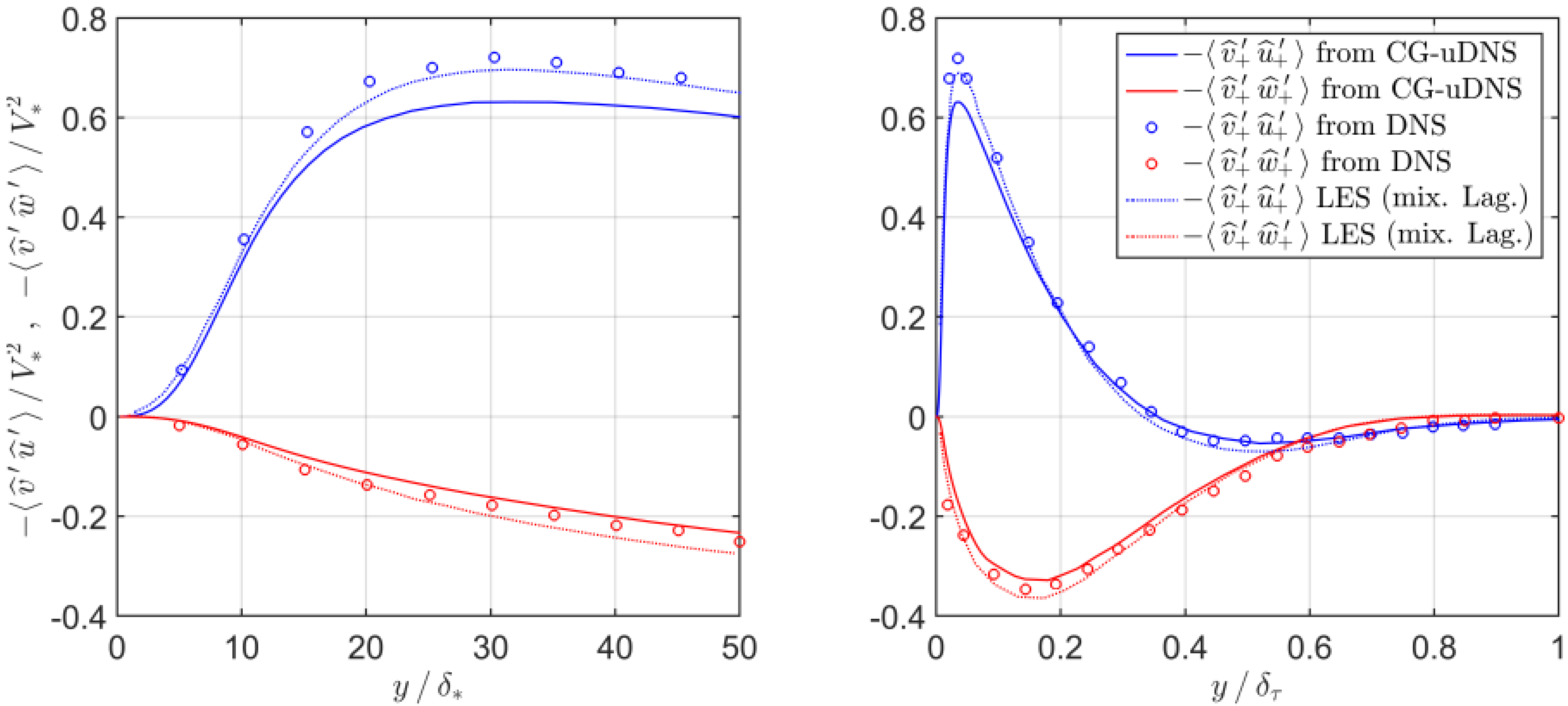
CG-uDNS (p=4) on 32^3 dofs VS. mixed Lagrangian model (FD-based) on 64^3 dofs



Imperial College London

Results of run 8p4

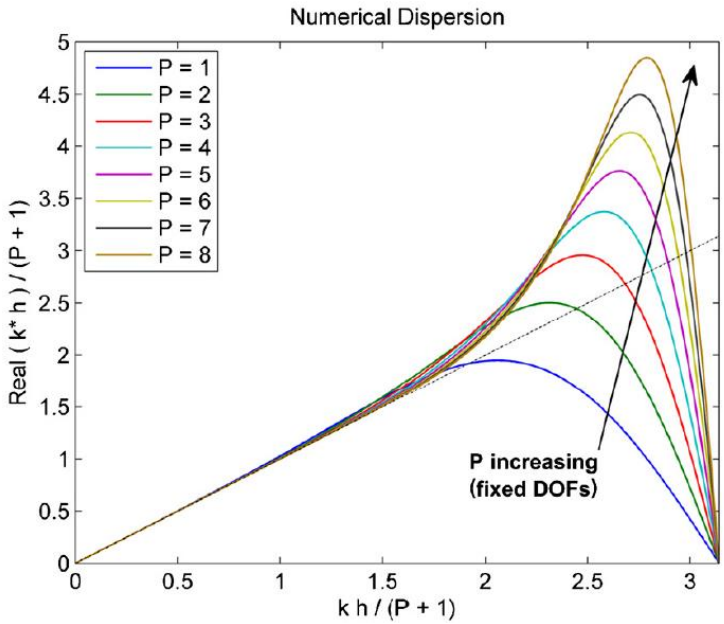
CG-uDNS (p=4) on 32^3 dofs VS. mixed Lagrangian model (FD-based) on 64^3 dofs

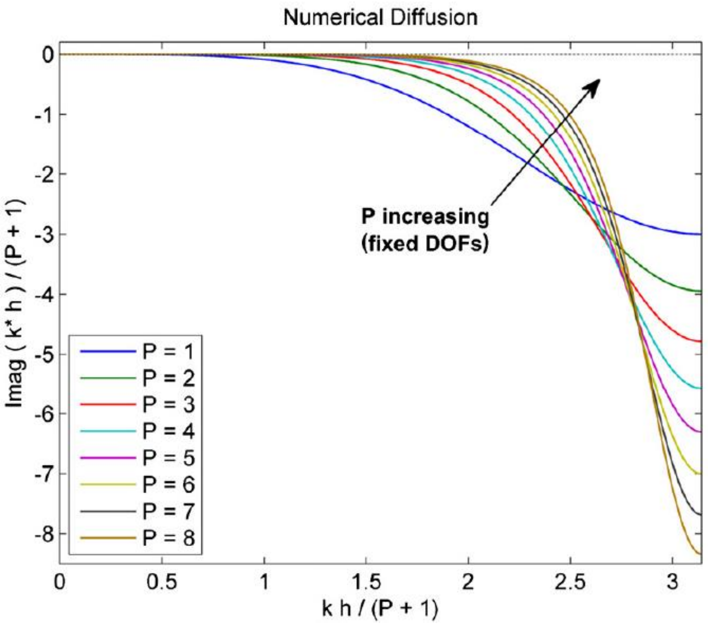


Imperial College London

Why so much success?

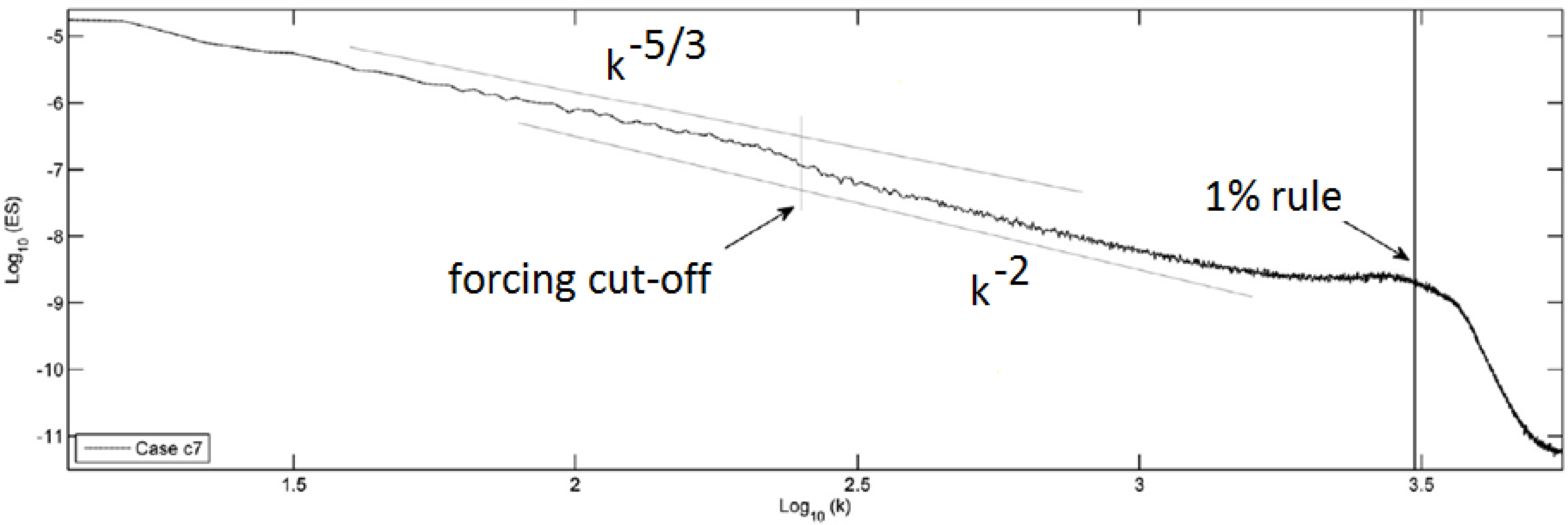
- Superior resolution power per DOF at higher orders
- Absence of (often restrictive) modeling assumptions
- Reynolds not too high yet (no SVV need so far)





Future directions

- Analysis of the energy spectrum, and other statistics
- Try and correlate with dispersion-diffusion analysis
- Increase the Reynolds number (and probably add SVV)



Imperial College London

Questions