From Hydrodynamic instability to chaotic mixing

with NEKTAR++

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Nektar++ Workshop 2019

Plan of the Presentation

Introduction
Rationale

- Pattern based flow control
- Mass and heat transfer intensification
- Cooling of microelectronics
- Blood oxygenations, DNA screening microarray
- Drag reduction and roughness modelling

What is mixing?

'If you have to ask what jazz is, you'll never know.' Louis Armstrong

- In fluids it is a two-stage process (Eckart, 1948)[1]
  - mechanical stirring
  - inter-material diffusion
- Stirring produces small scales (layers) in a stretching and folding action much like the horseshoe transformation, that can be rapidly smoothed by diffusion
- Turbulization is effective, but not always applicable

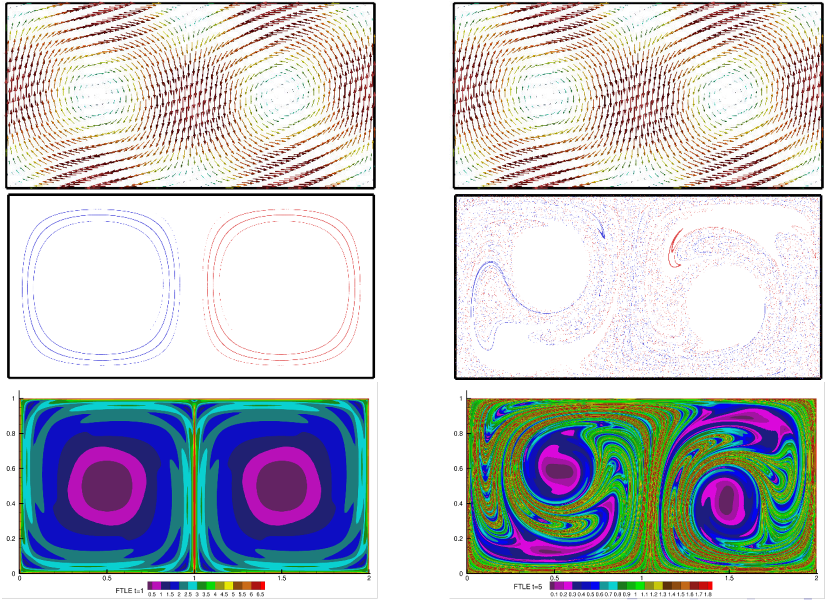
[1] Eckart, C. 1948. An Analysis of the Stirring and Mixing Processes in Incompressible Fluids. J. Mar. Res., 7, 265–275.

Chaotic advection and mixing

- Simple, low Reynolds number flow lead to the onset of Lagrangian chaos (Aref 1984)[1]
- Motion described by $\dot{\mathbf{x}} = \mathbf{u}(\mathbf{x}, t)$
- Or $\dot{x} = \partial\Psi/\partial y, \quad \dot{y} = -\partial\Psi/\partial x$, equivalent to a single-degree-of-freedom system
  1. Two dimensional, steady flows result in integrable advection equations meaning that particle trajectories are regular.
  2. Unsteady flows in two dimensions and steady or unsteady flows in three dimensions may result in non-integrable advection equations leading to chaotic particle trajectories.

[1] Aref, Hassan. 1984. Stirring by chaotic advection. Journal of Fluid Mechanics, 143, 1–21.

Double gyre example



Quantifying mixing

Is not well agreed upon

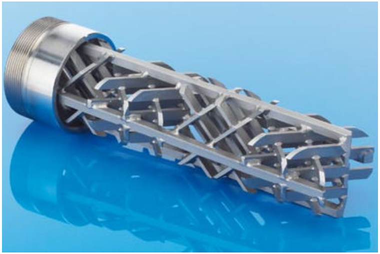
- Strange eigenmodes - eigen function to the advection-diffusion process. Leading AD operator mode corresponds to the slowest decaying initial distribution of the advected scalar. This mode is a representation of structures that are persistent under the flow, with the corresponding eigen value determining the exponential decay rate. Consequently, it imposes an upper limit on the efficiency of the stirring
- Dynamical system analogy (Ottino 1989)[1]
  - strives to quantify the stirring action by the amount of stretching experienced by individual fluid parcels traced in the advection field
  - based on the notion of the specific rate of stretch
- Norms[2]: L_2 and H^{-1}
  - L^2: Variance $Var\theta = \frac{1}{|\Omega|}\int_\Omega \theta^2 d\Omega)$
  - Mix-norm based on negative index Sobolev norms

[1] Ottino, Julio M. 1989. The kinematics of mixing: stretching, chaos, and transport. Vol. 3. Cambridge university press.

[2] Thiffeault, Jean-Luc. "Using multiscale norms to quantify mixing and transport." (2012).

How to invoke chaotic advection without turbulization?

- Baffles, obstacles, bends
- External actuation
- Why not hydrodynamic stability?
- Optimal initial perturbation in Poiseuille flow (Vermach 2018[1], Foures 2014[2])



Mixers for the Plastics Processing Industry, Sulzer Chemtech, https://sulzer.com/

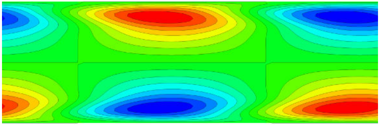
[1] Foures, D., Caulfield, C., & Schmid, P. (2014). Optimal mixing in two-dimensional plane Poiseuille flow at finite Péclet number. Journal of Fluid Mechanics, 748, 241-277.

[2] Vermach, L., & Caulfield, C. (2018). Optimal mixing in three-dimensional plane Poiseuille flow at high Péclet number. Journal of Fluid Mechanics, 850, 875-923.

What everybody knows

about hydrodynamic stability

- We look for eigenfunctions of the linearised NS operator
- Those could be either attenuated or amplified, stationary or travelling
- In the smooth channel case the critical perturbation is the 2D TS wave that becomes unstable at $Re_{cr} = 5772, \delta = 1.02$ and travels downstream with frequency $\sigma_r \approx 0.27$ and phase speed $v_p = \sigma_r/\beta_{cr} \approx 0.26$



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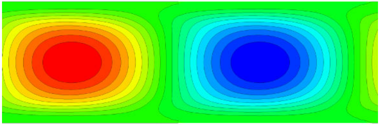
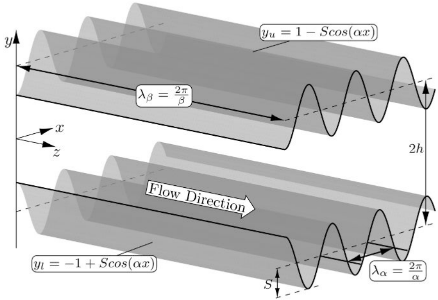


v:

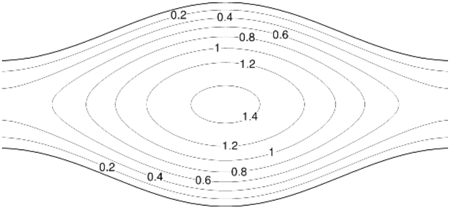
Longitudinal grooves

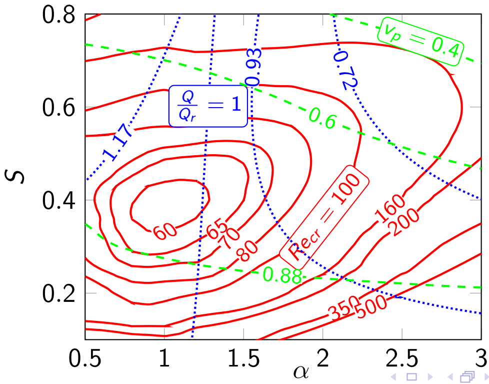


- S - corrugation amplitude
- $Re = \frac{UL}{\nu}$ - reference flow, $Q_r = \frac{4}{3}$
- n - number of corrugations in computations
- α - spanwise wave number $\rightarrow \lambda_\alpha = \frac{2\pi}{\alpha}$
- β - streamwise wave number

$$(\alpha, S, Re, n, \beta)$$

2D base flow and stability





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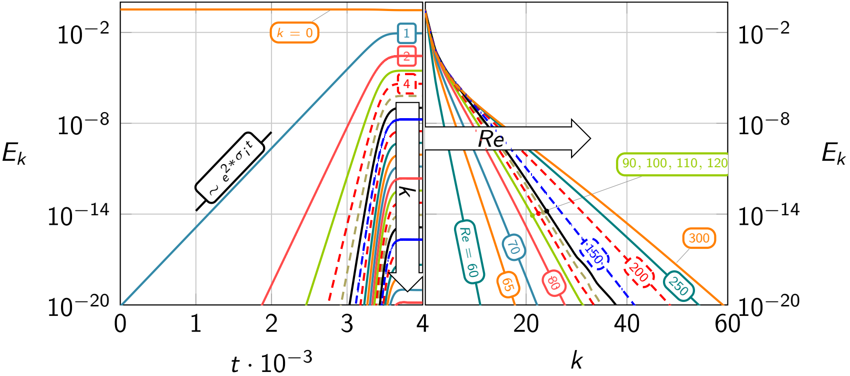
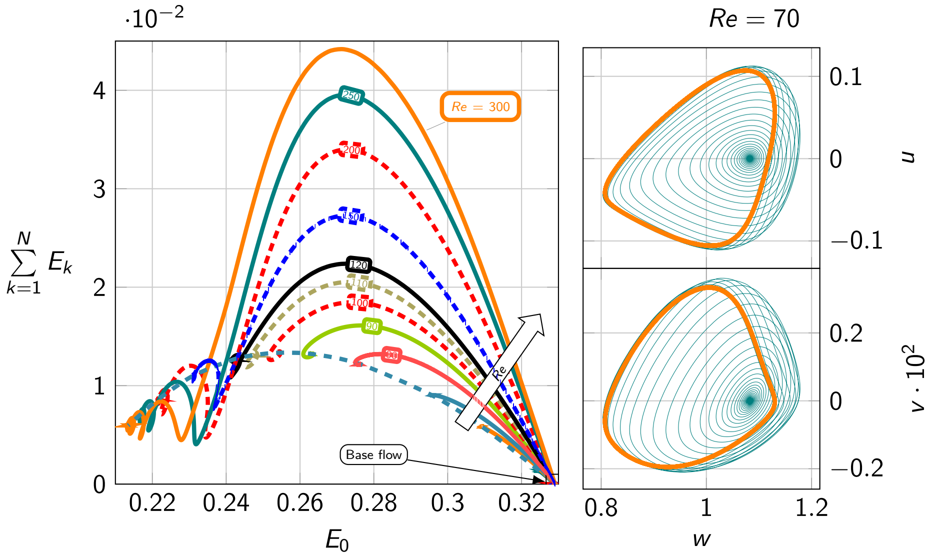
Nonlinear saturation

$\alpha = 1, S = 0.4, Re = 70, \alpha = 1, \beta = 0.4$



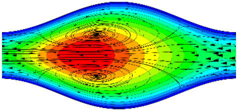
Nonlinear Saturation

$\alpha = 1, S = 0.4, \alpha = 1, \beta = 0.4$

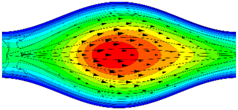


Nonlinear Saturation

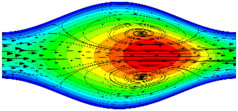
Flow pattern $\alpha = 1, S = 0.4, Re = 80, \alpha = 1, \beta = 0.4$



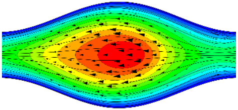
$z = \frac{\lambda_\beta}{4}$



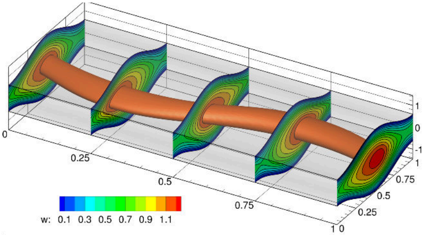
$z = \frac{\lambda_\beta}{2}$

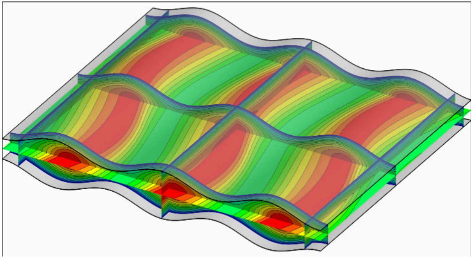


$z = \frac{3\lambda_\beta}{4}$



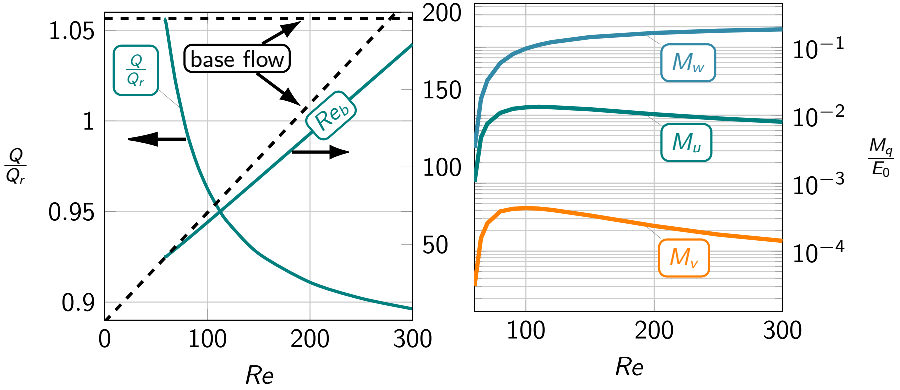
$z = \lambda_\beta$





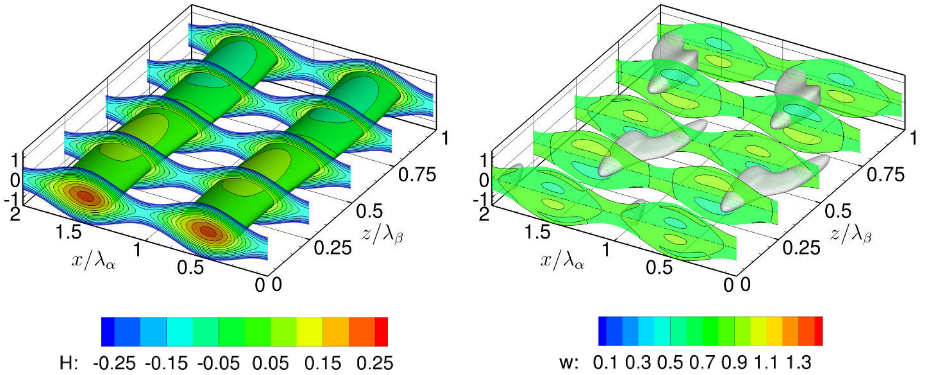
Nonlinear Saturation

$$M_q = \frac{1}{\Omega} \int_\Omega (q_s - q_b)^2 d\Omega$$



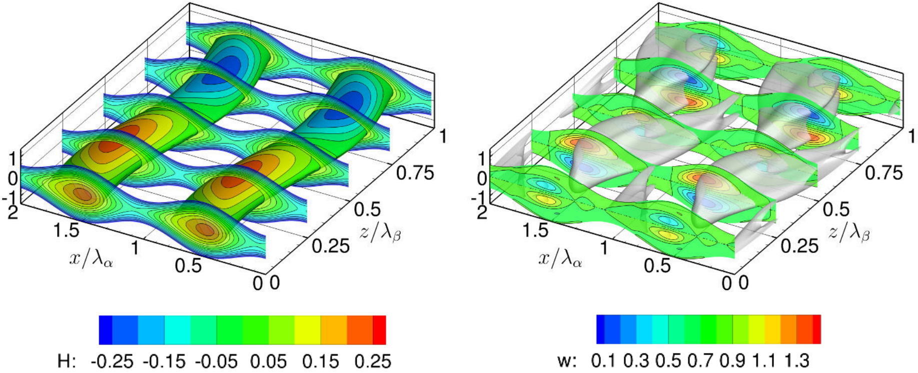
Nonlinear Saturation

$Re = 60$



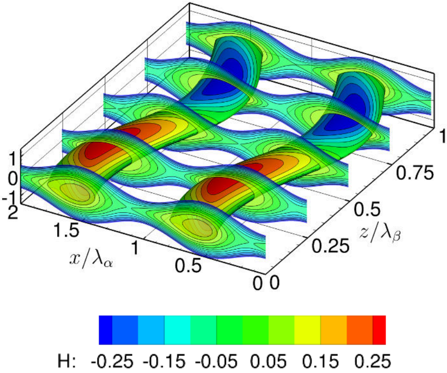
Nonlinear Saturation

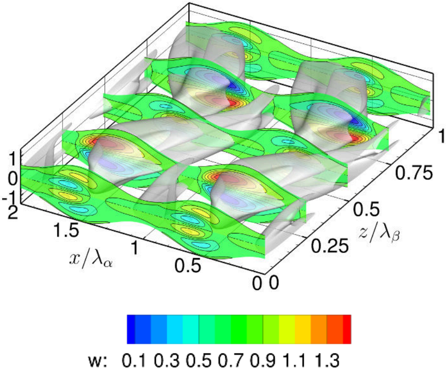
$Re = 70$



Nonlinear Saturation

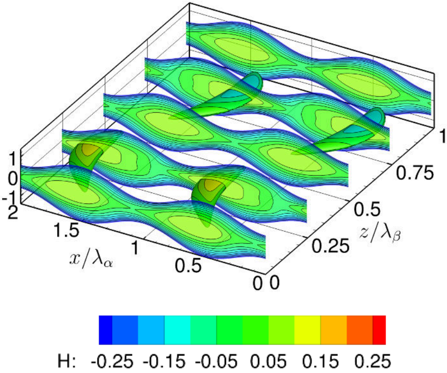
$Re = 100$

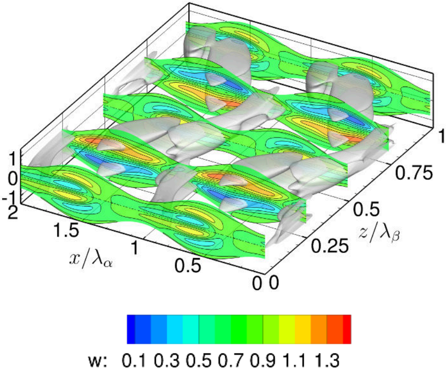




Nonlinear Saturation

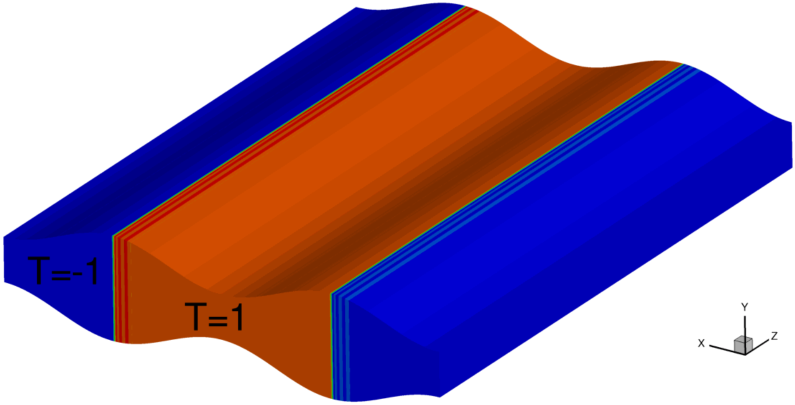
$Re = 250$





Quantification of mixing

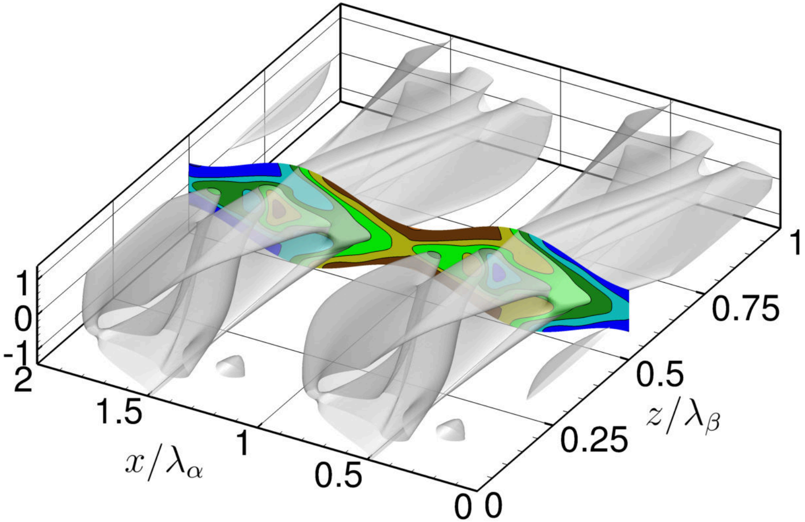
Homogenization of a passive scalar



Quantification of mixing

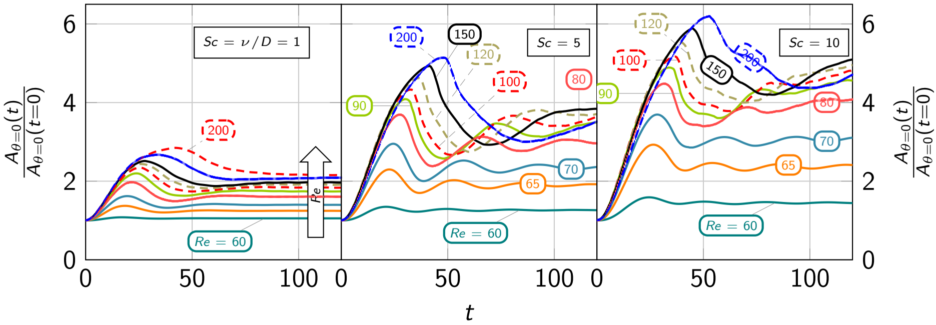
Homogenization of a passive scalar

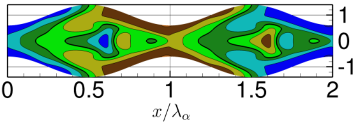
$Re = 100,\ Sc = 10,\ \theta = 0$

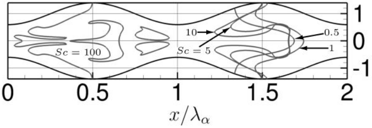


Interface area

Homogenization of a passive scalar

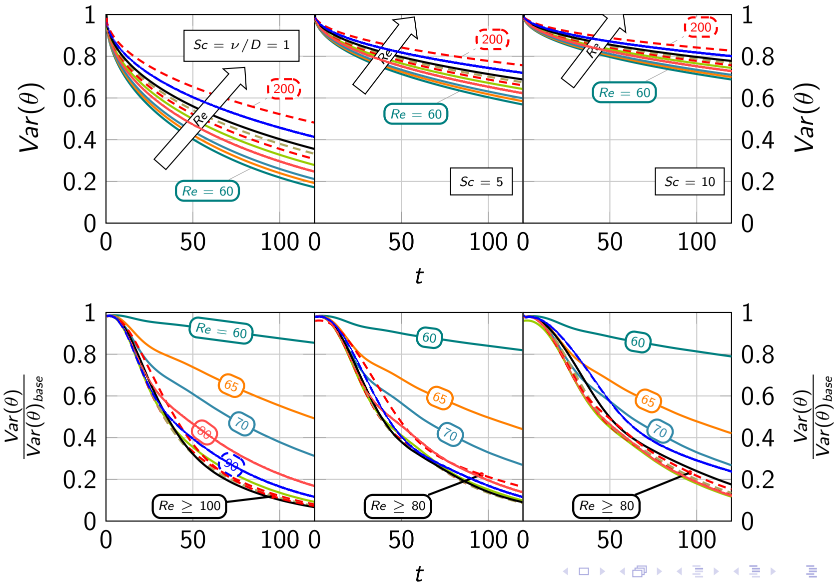






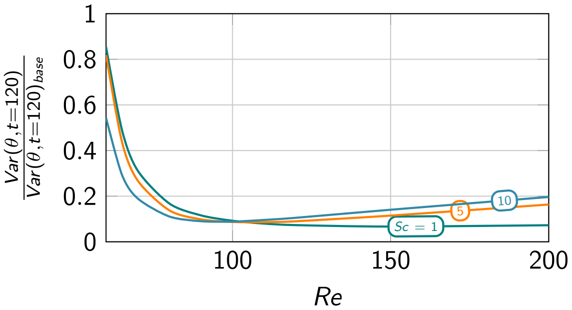
Quantification of mixing

Homogenization of a passive scalar



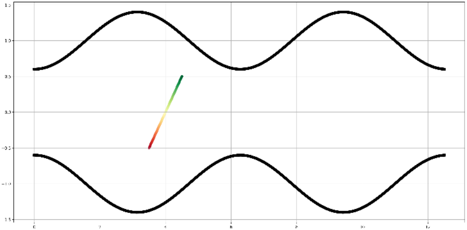
Quantification of mixing

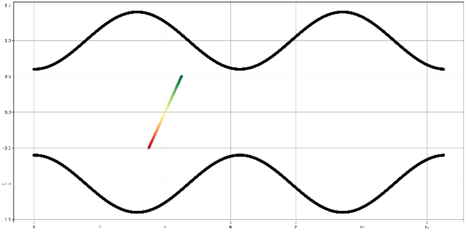
Homogenization of a passive scalar



Poincaré sections

$L = 320h$, **intersections every** $10h$





Poincaré sections

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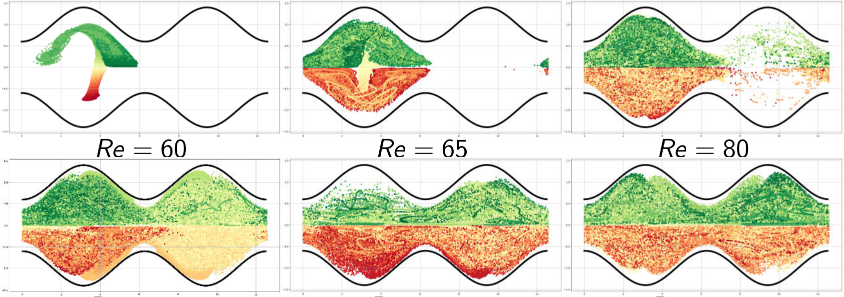
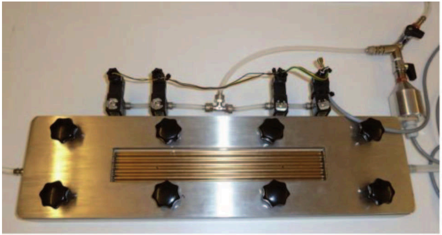
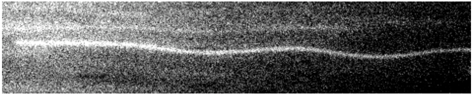


$Re = 60$ $Re = 65$ $Re = 80$

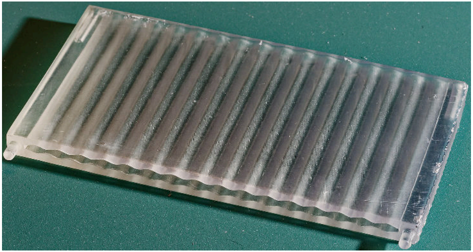
$Re = 100$ $Re = 150$ $Re = 200$

Experiment





$L = 0.3mm, \; t = 0.12s, \; Re = 120$



[0] Szumbarski, Jacek, Blonski, Slawomir, & Kowalewski, Tomasz. 2011. Impact of transversely-oriented wallcorrugation on hydraulic resistance of a channel flow. Archive of Mechanical Engineering, 58(4), 441.