Thermal convection with flow rate forcing in Nektar++

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Motivation

Currently Nektar++ [1], an open-source spectral/hp element software, is not configured for the solution of thermal convective flow with flow rate forcing. This forcing acts as a means to sustain the forced convective flow.

Governing equations

Navier-Stokes equations:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f_b} \qquad (1)$$

Advection-Diffusion equation:

$$\frac{\partial \theta}{\partial t} + \boldsymbol{u}\cdot\nabla\theta = \epsilon\nabla^2\theta \qquad (2)$$

where, $\boldsymbol{u}$ (u,v,w) is the velocity vector, p is the specific pressure, θ is the temperature, ν is the kinematic viscosity, ϵ is the diffusion coefficient.
The vertical body force term (due to heating)

$$f_b = \frac{Ra\,\theta}{Pr\,Re^2}$$

Ra – Rayleigh number,
Pr – Prandtl number,
Re – Reynolds number.

Computational methods

- Explicit treatment of the body force f_b in Navier-Stokes equations decouples the Advection-Diffusion equation.
- Momentum equations are solved using higher order velocity and pressure splitting scheme by Karniadakis, Israeli and Orszag [2], and Guermond and Shen [3]. This scheme essentially treats the advection (non-liner) terms explicitly, solves the pressure Poisson system, and finally solves a Helmholtz problem using pressure field to enforce viscous forcing and boundary conditions.
- In addition to the thermal forcing, flow rate forcing is used as an external forcing.
- To enforce the flow rate forcing Green's function method [4] is used. At the first-time step, Stokes equation with unit flow rate is solved to obtain an intermediate velocity field [5].
- For the Stokes solver, presence of the explicit term f_b causes the pressure Neumann boundary condition to be non-zero. Since Stokes solver requires homogeneous boundary condition, at the first time step this pressure boundary condition is explicitly set to zero.
- For time integration, second order implicit-explicit (IMEX2) scheme is used.

Test cases

The following test cases are considered,

(i) 2D channel flow with uniform heating at bottom wall:

- Prescribed initial conditions:

  $u = 1 - y^2,\ v = 0,\ P = \frac{Ra}{2\,PrRe^2}\left(y - \frac{y^2}{2}\right),\ \theta = (1-y)/2$
- Comparing with the exact solution for *Re* = 1 to 100 and *Ra* = 0 to 1000, maximum error norm in velocity fields obtained is ~10^{-12}.

(ii) 2D channel flow with uniform heating at upper wall:

- Comparing with the exact solution, maximum error norm in velocity fields obtained is also ~10^{-12}.

(iii) 2D channel flow with periodic heating at bottom wall:

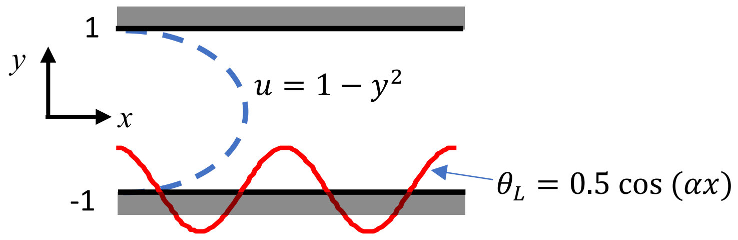

Figure 1: Channel flow with periodic heating. α – heating wave-number.

- The obtained solution is compared with a spectral solver [6], and the velocity and temperature fields agree at least four digits.
- Figure 2 shows the velocity and temperature fields which are in qualitative agreement with the solver used in [6].

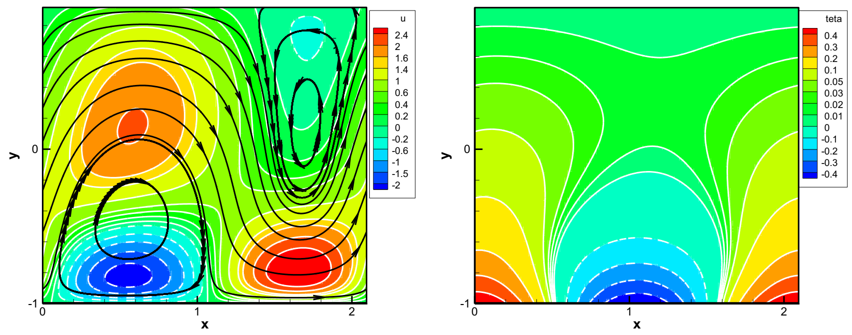

Figure 2: u-velocity distribution and streamlines (left) and temperature (θ) distribution (right) for Re = 1, Ra = 600, α = 3.

(iv) 3D1H channel flow with uniform heating at bottom wall: (homogeneous in z-direction).

- Conditions in case (*i*) together with w = 0 are prescribed.
- It is observed that the 2D solutions obtained in case (*i*) are recovered.

Conclusion and future direction

Various test cases suggest that the code provides expected accuracy.
The followings are some of the future directions,

- flow physics for 3D1H and 3D cases to be captured,
- flow stability of the above cases to be examined using the stability solver.

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