Destabilisation and modification of Tollmien-Schlichting disturbances by a 3D surface indentation

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Highlights

The onset of laminar-turbulent transition from a confined thin 3D separation bubble:

- Identify the mechanisms of destabilisation and modification of spanwise-uniform 2D TS modes
- Topological shapes of separation bubbles
- Inflectional mechanism
- Emergent non-linear flow phenomena
- Success or failure of transition criteria

Motivation

Laminar-turbulent transition in boundary layers is a fundamental topic, which poses a considerable theoretical and numerical challenge.

1. Two kinds of problems: (i) receptivity mechanism; (ii) stabilisation/destabilisation of TS waves. A small part of possible receptivity & stabilisation/ destabilisation mechanisms was investigated.
2. Studies of the interaction between instability modes and a distorted base flow have received less attention.
3. Destabilisation and modification of spanwise uniform 2D TS modes in a boundary layer with a confined thin separation bubble.
4. Emergent onset of transition induced by a confined thin separation bubble.

Mathematical formulations

The fully nonlinear Navier-Stokes equations (NSEs) under a non-deforming mapping from Ω' to Ω:

1. $x = x',\ y = y' - \zeta(x', z'),\ z = z'$.
2. $u = u',\ v = v' - u'\partial_{x'}\zeta - w'\partial_{z'}\zeta,\ w = w', p = p'$.
3. $\partial_{x'} \equiv \partial_x - \zeta_x\partial_y,\ \partial_{y'} \equiv \partial_y,\ \partial_{z'} \equiv \partial_z - \zeta_z\partial_y$.
4. The NSEs are transformed as follows (in Ω)

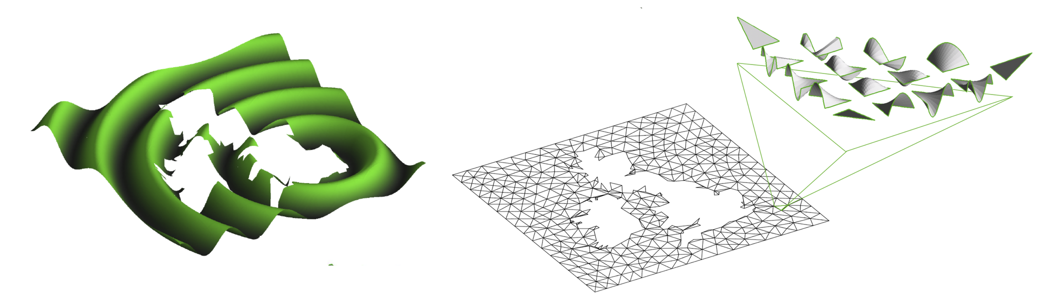
$$\begin{cases} \partial_t u_i - Re^{-1}\partial_j^2 u_i + u_j\partial_j u_i + \partial_i p = \mathcal{A}_i \\ \partial_j u_j = 0. \end{cases}$$

5. $\zeta(\cdot,\cdot)$ is defined by $(r^2 = (x' - x'_c)^2 + (z' - z'_c)^2)$

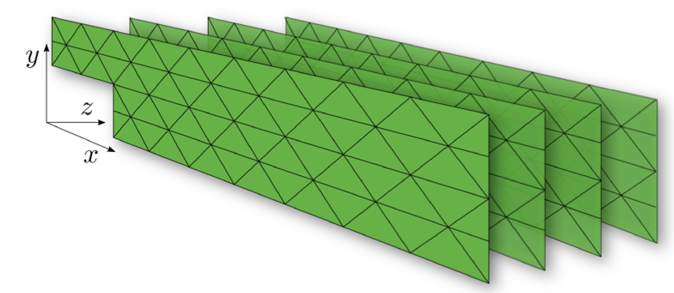
$$\zeta(x,z) = \begin{cases} -\dfrac{h}{2}\cdot\left(\cos\left(\dfrac{2\pi\cdot r}{\lambda}\right)+1\right), & r \le \lambda/2, \\ 0, & r > \lambda/2, \end{cases}$$

Methodology

- Spectral/*hp* method in Nektar++



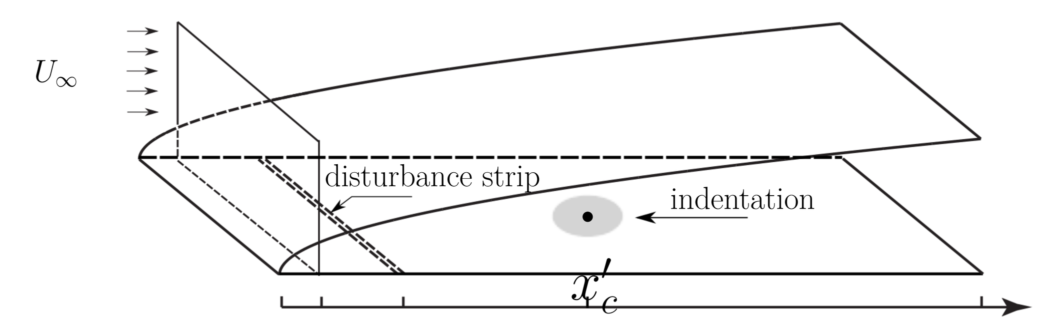
- Fourier expansion in z-direction & spectral element discretisation in x-y plane



- Global mapping technique

Configurations & Parameters

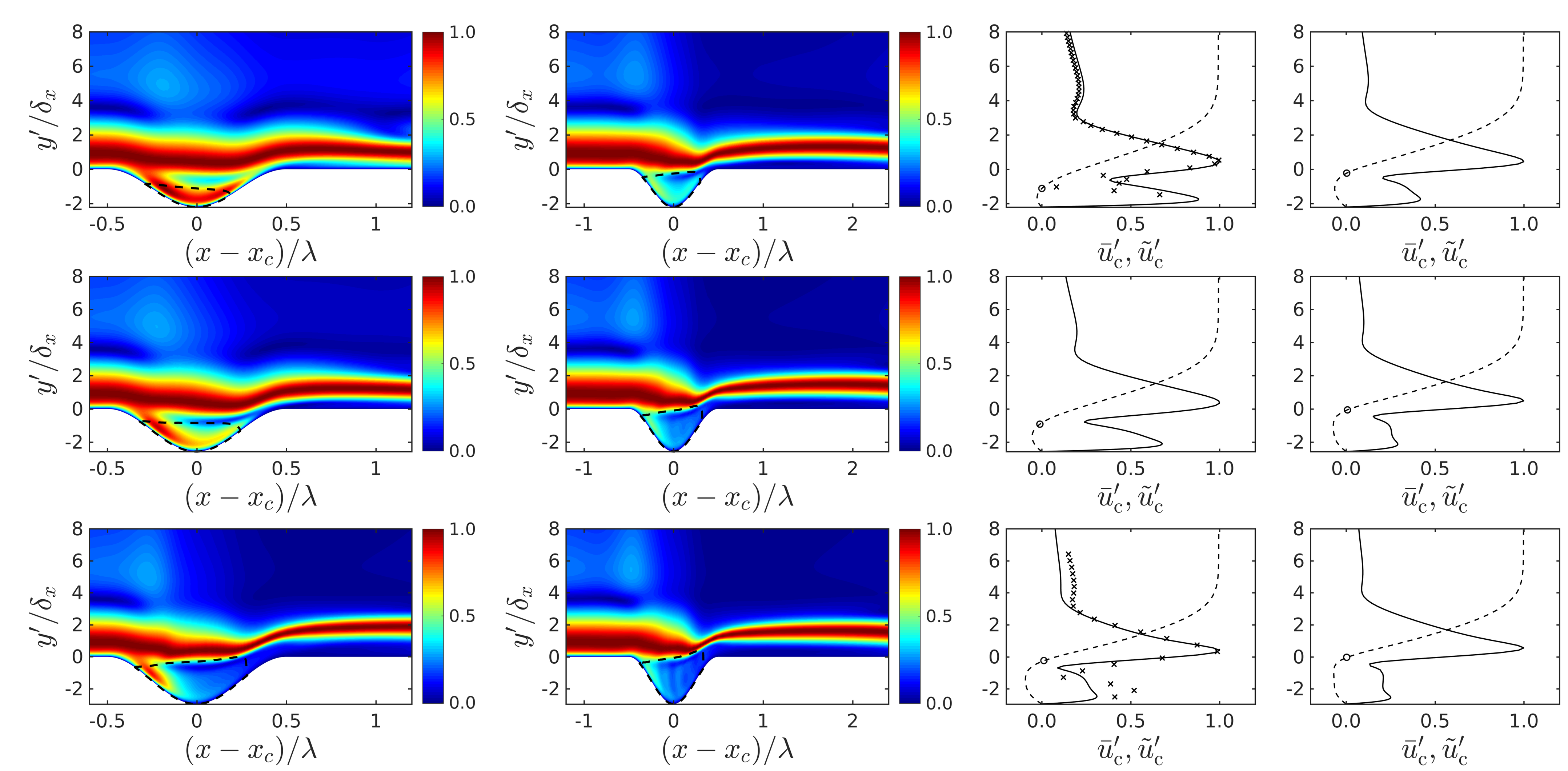
- Freestream Re_f/m=1.2×10^6/m & U_∞=18m/s
- $\Omega^1 = [0.1, 1.1] \times [0, 0.05] \times [-0.20, 0.20]$(m)
  $\Omega^2 = [0.1, 1.1] \times [0, 0.05] \times [-0.08, 0.08]$(m)



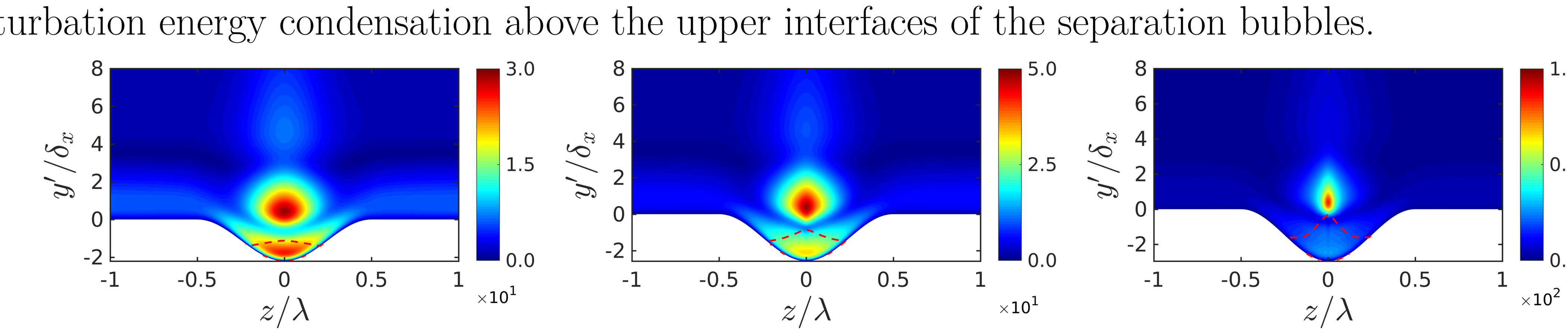
x'_c(m)	$\mathcal{F}$(Hz)	λ^1(mm)	λ^2(mm)	$h^{1,2}$(mm)
0.649	172	81	40.5	1.620
—	—	—	—	1.895
—	—	—	—	2.170

Topology of Separation Bubbles & Destabilisation of Instability Modes

1. Topology shapes of separation bubbles strongly depends on h & λ.
2. Destabilising and modifying incoming TS modes' profiles.



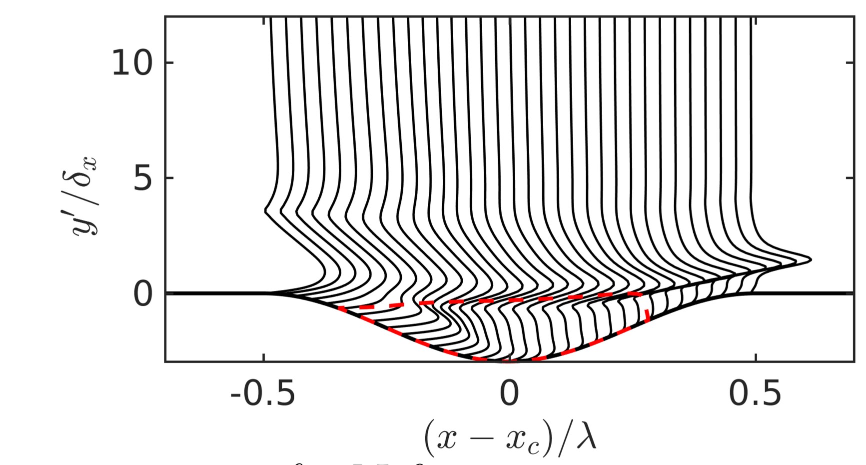
3. In the planes $x' = x'_c$, the separation bubble upper surface develops a cusp structure.
4. Perturbation energy condensation above the upper interfaces of the separation bubbles.



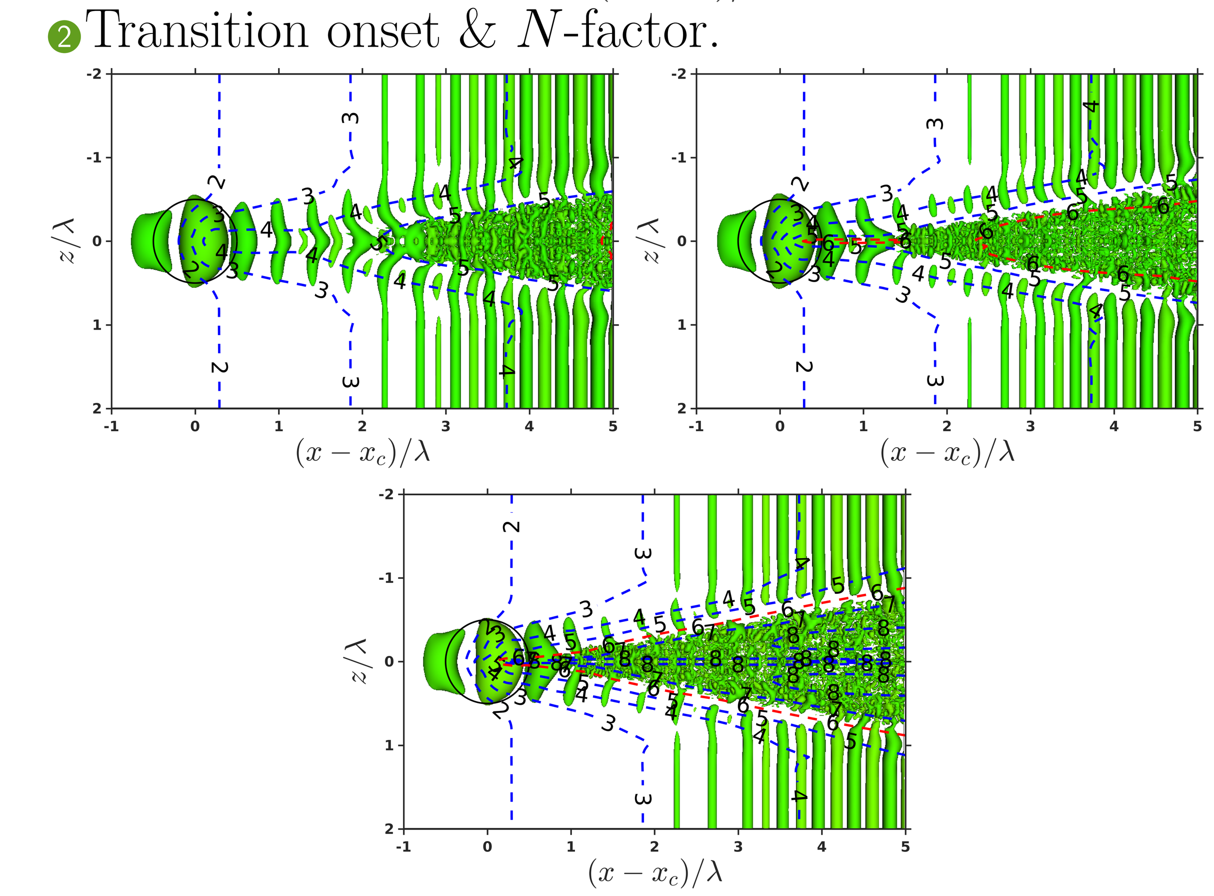
5. Deeper indention gives rise to larger amplification of the TS modes and a sharper cusp structure.

Mechanism & Transition Onset

1. Inflectional mechanism.



2. Transition onset & N-factor.



Conclusion

1. A cusp-like structure develops on the upper interfaces of the separation bubbles.
2. The cusp structure energises instability modes concentrated at the tip of the cusp.
3. The primary instability mechanism in a thin separation bubble is inflectional in nature.
4. A shallow indentation can make the onset prediction of the traditional criteria fail.

Acknowledgements

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Information

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- *Xu, et al, JFM*, 819: 592-620, 2017.