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Two-dimensional high-order mesh generation and r-adaption using a variational framework

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Motivations

High-order CFD methods are highly susceptible to inaccuracies in the geometrical representation of meshes. The core of our research focuses on the generation of CAD-accurate high-order meshes for curvilinear geometries. The work presented below concerns recent developments in the open-source code **NekMesh** relative to two-dimensional mesh generation.

A new approach to r-adaption is also presented. This method is based on a variational framework initially developped for mesh optimisation, that allows to move both vertex and high-order nodes using a local and highly scalable approach.

High-order mesh generation

In the framework of **NekMesh**, high-order meshes are generated in two steps [1]:

The first step consists in generating a linear mesh with an outside-in approach. First, curves are discretised using a spacing based on local curvature; then, triangulation is applied to surfaces.

The second step deals with the high-order nodes which are first generated on the linear elements before being projected onto the CAD curves and surfaces. This second step can in fact be applied to a mesh that was not generated internally.

Two-dimensional mesh generation features

- STEP file format support.
- NACA aerofoil geometry generation.
- Structured boundary layer mesh generation: based on the splitting of a single boundary layer element as shown in [2]. The boundary can be closed or open.

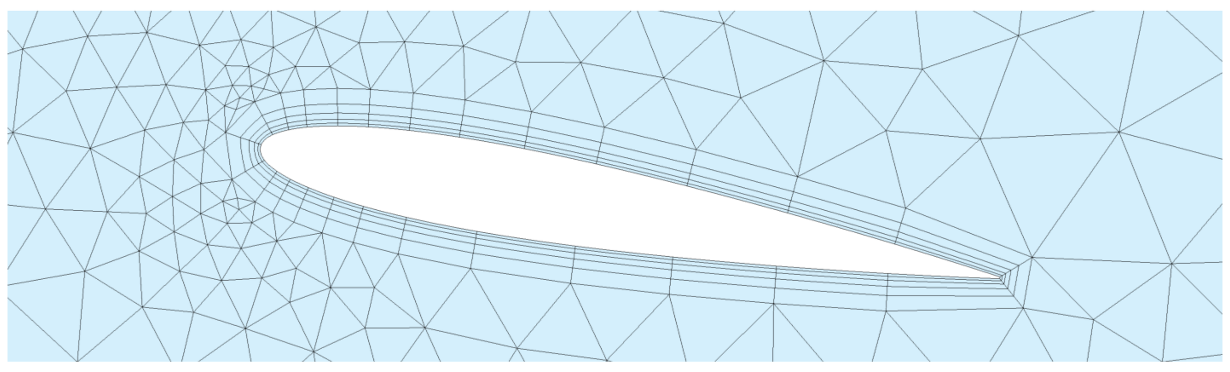

Figure 1 : Mesh of a NACA 0012 aerofoil at 10° angle of attack including a structured boundary layer, generated with the NACA generator.

- GEO file format support: standard used by Gmsh.
- Conforming boundary mesh for periodic boundary conditions.

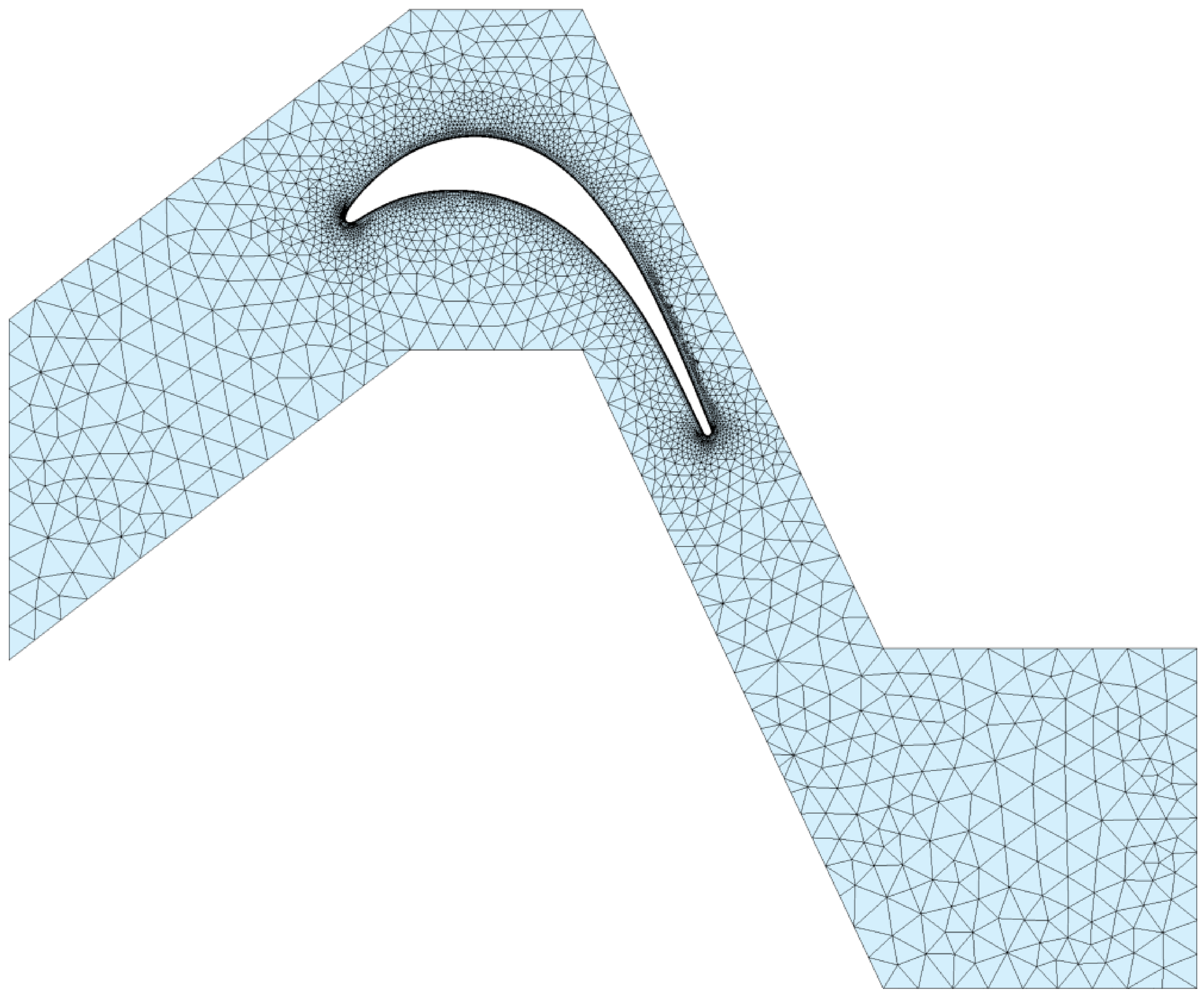

Figure 2 : Mesh of a T106a blade profile with periodic boundaries and boundary layer mesh, generated from a GEO file (courtesy of Andrea Cassinelli).

Variational optimiser

High-order mesh optimisation in **NekMesh** is performed using a variational framework. The mesh is deformed to minimise an energy functional ε, a function of the mesh deformation ϕ [3]:

$$\text{find } \min_{\phi} \varepsilon(\nabla\phi) = \int_{\Omega^e} W(\nabla\phi) dy$$

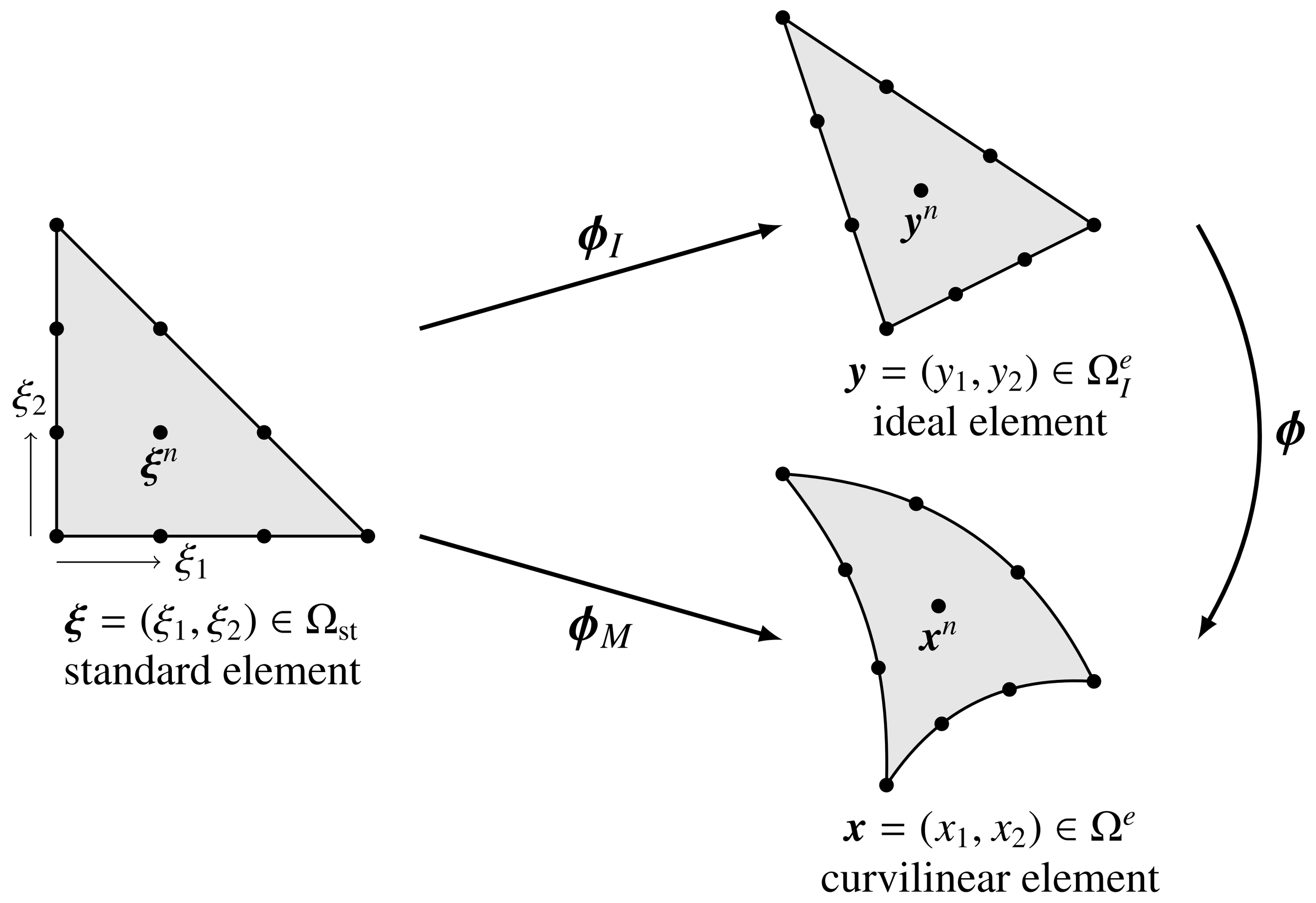

Figure 3 : Existing mappings between the reference, the straight-sided and the curvilinear elements.

r-adaption

To aphieve r-adaption, the mapping ϕ_I used in the variational framework is modified to force the deformation of individual elements. This way, we can obtain anisotropic mesh deformation based on a tensor metric field. Furthermore, the mapping ϕ_I can be spatially varying inside each element leading to finer adaption in non-smooth metric fields. Finally, because **NekMesh** retains all CAD information, nodes are able to slide on boundaries while retaining full geometrical accuracy.

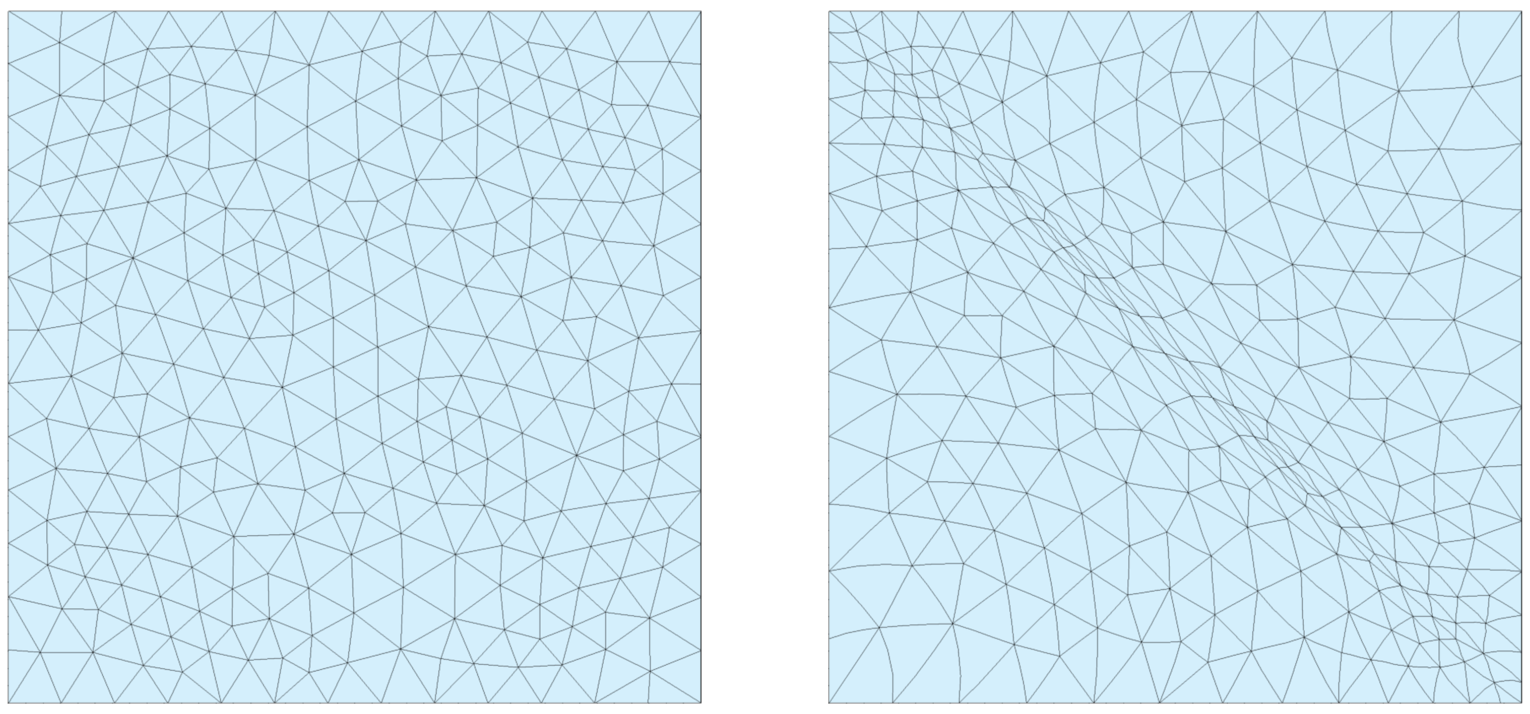

Figure 4 : Anisotropic adaption along a diagonal: initial (left) and deformed meshes (right)

Summary

The work hereby presented is part of ongoing developments made in the open-source code **NekMesh** and is intended in aiding high-order CFD, including **Nektar++** [4], users in their meshing tasks. The two-dimensional mesh generator already provides features covering a large array of meshing needs and current research and development efforts are focused on the varionational approach to r-adaption. Future work will focus on sensors and metrics and their application to r-adaption.

Acknowledgements

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