# Various types of hydrodynamic instability in a channel

## with NEKTAR++

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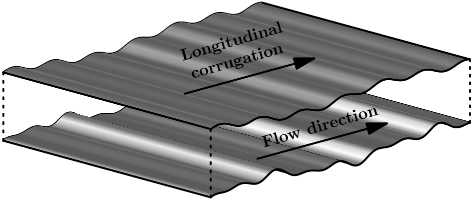
# Plan of the Presentation

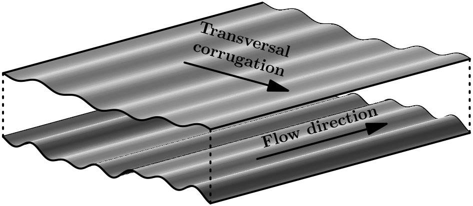
# Introduction
## Rationale

- Blood oxygenations
- Mass and heat transfer
- Piezoelectric energy harvesting devices
- Sand accumulation - brick-pattern ripples
- Study of the onset of oscillatory and aperiodic states
- Drag reduction and roughness modelling
- Transverse fluid motion

# Introduction
## Today

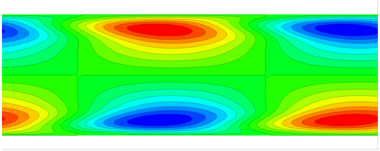
- Influence of large scale corrugation
- Three types of unstable modes
  - Travelling and stationary
  - Tollmien-Schlichting like and Squire like modes
- Only linear stability and normal modes today (no transient phenomena, no bypass)
- Some results illustrating saturation of the unstable modes

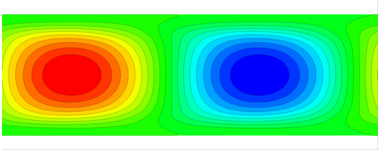


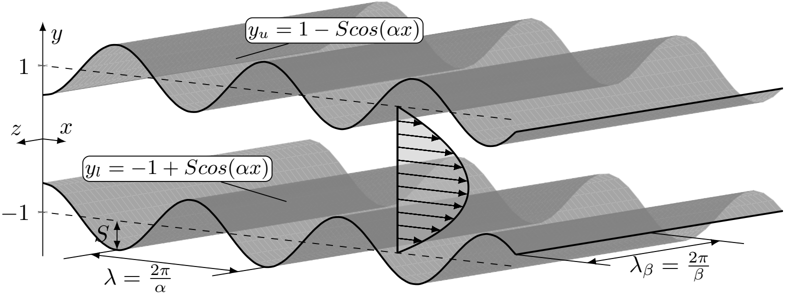
# What everybody knows

- We look for eigenfunctions of the linearised NS operator
- Those could be either attenuated or amplified (amplification), stationary or travelling (waves - phase speed)
- In the smooth channel case the critical perturbation is the 2D TS wave that becomes unstable at $Recr = 5772, \delta = 1.02$ and travels downstream with frequency $\sigma_r \approx 0.27$ and phase speed $v_p = \sigma_r/\beta_{cr} 0.26$

u: 

v: 

# Longitudinal corrugation

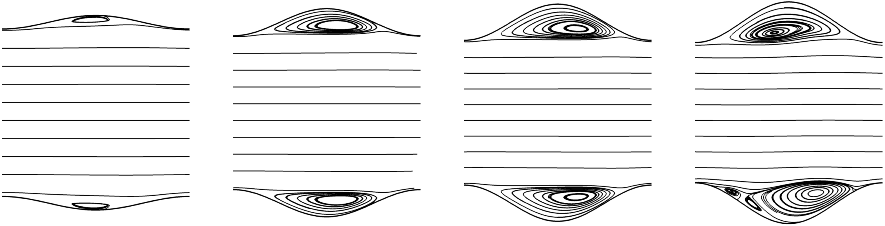



- $\alpha$ - corrugation wave number $\rightarrow \lambda = \frac{2\pi}{\alpha}$
- $S$ - corrugation amplitude
- $Re = \frac{UL}{\nu}$ - reference flow, $Q_r = \frac{4}{3}$
- $n$ - number of corrugations in computations
- $\beta$ - spanwise number $\rightarrow \lambda_\beta = \frac{2\pi}{\beta}$
- $\delta$ - travelling wave wave number

$(\alpha, S, Re, n, \mu, \delta)$

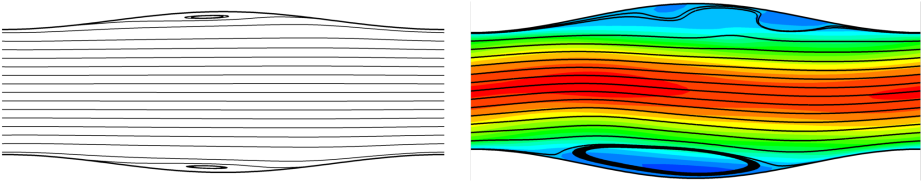
# 2D flow

$\alpha = 3$, $Re = 2300$



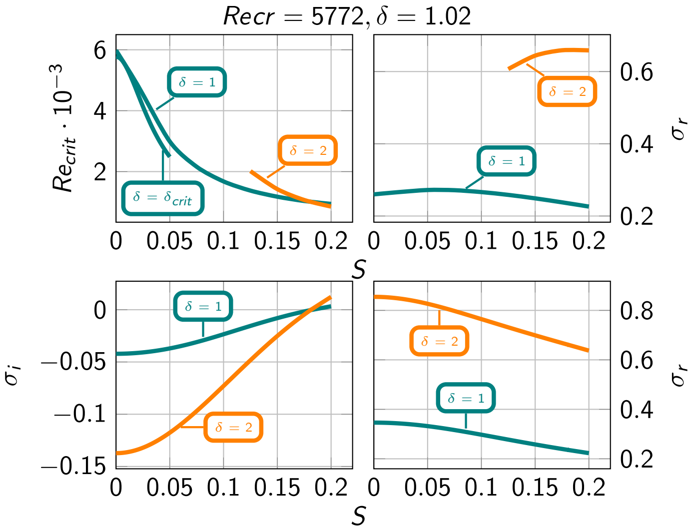
S=0.075 S=0.15 S=0.2 S=0.225

$\alpha = 1$, $S = 0.2$, $Re = 1000$



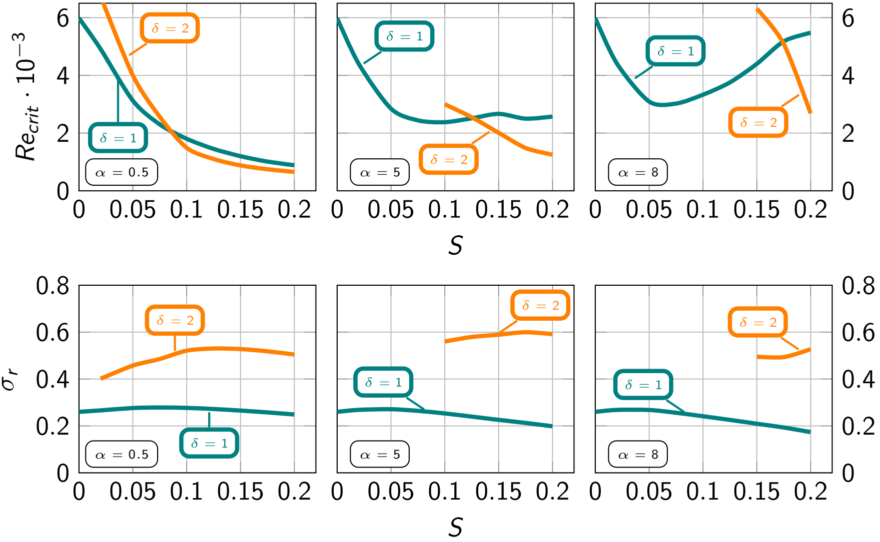
# 2D flow

## 2D travelling wave - $\alpha = 1, n = 1 \to \delta = 1, 2$



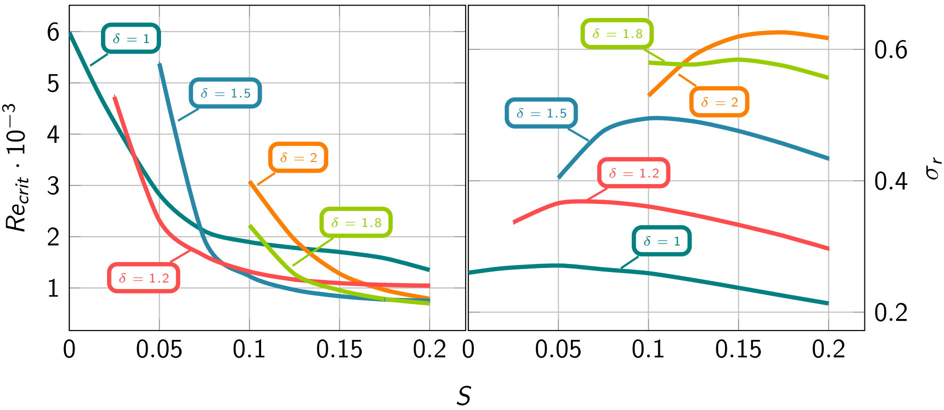
# 2D flow

## 2D travelling wave - $n = max(\alpha, 1) \rightarrow \delta = 1, 2$



# 2D flow

## 2D travelling wave - $\alpha = 3, n = 2, 3, 5, 7 \rightarrow \delta = 1, 1.2, 1.5, 1.8, 2$



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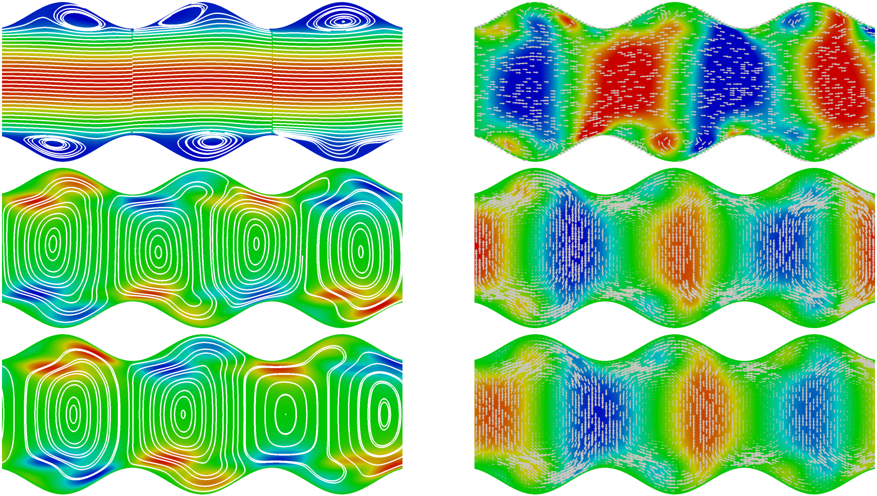
# 2D flow

## Transition to oscillatory flow - $\alpha = 3, n = 3 \rightarrow \delta = 1, 2$, $Re = 900$, $S = 0.2$

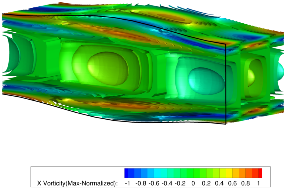
Ghaddar 1986 - Tollmien-Schlichting waves forced by Kelvin-Helmholtz shear-layer instability

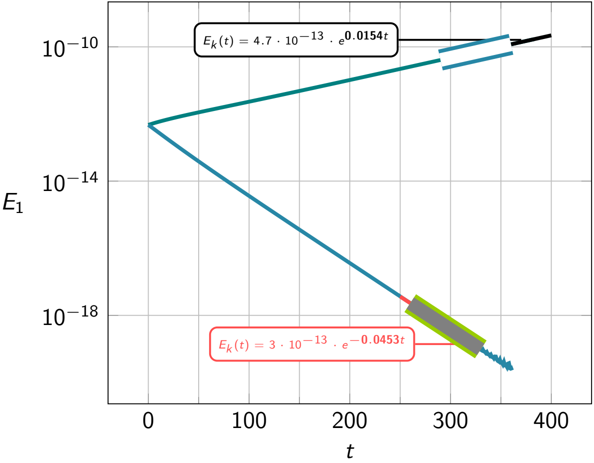


# 3D flow

## 3D stationary vortex

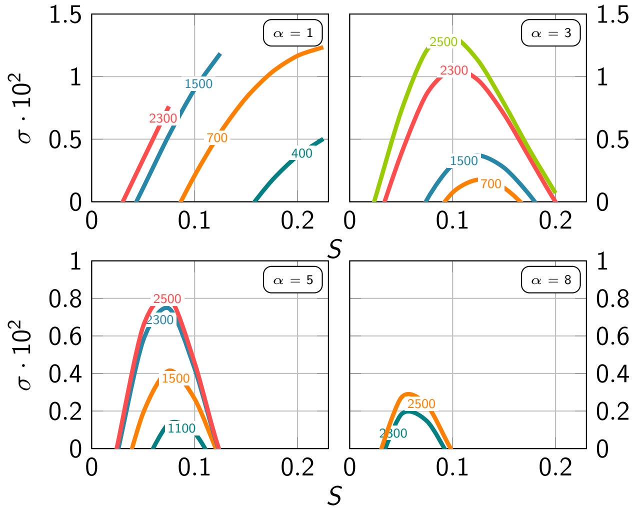
1. Expand base flows in the spanwise direction
2. Superimpose selected perturbation
3. DNS tracking the perturbation growth



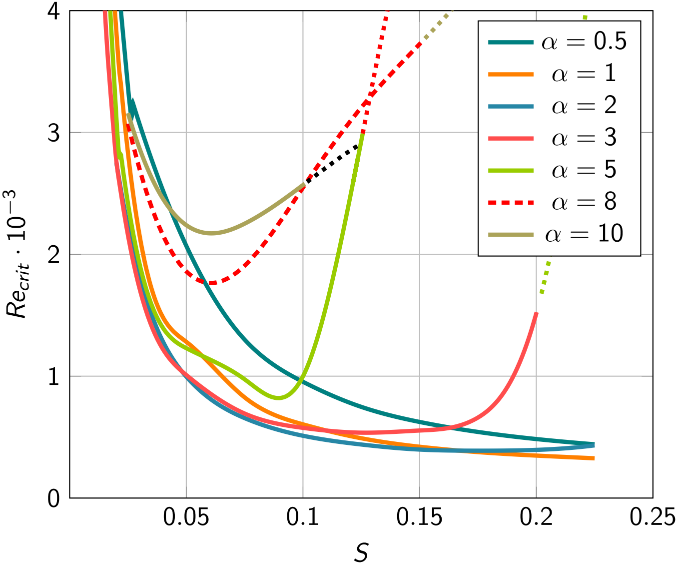
# 3D instability

## 3D stationary vortex

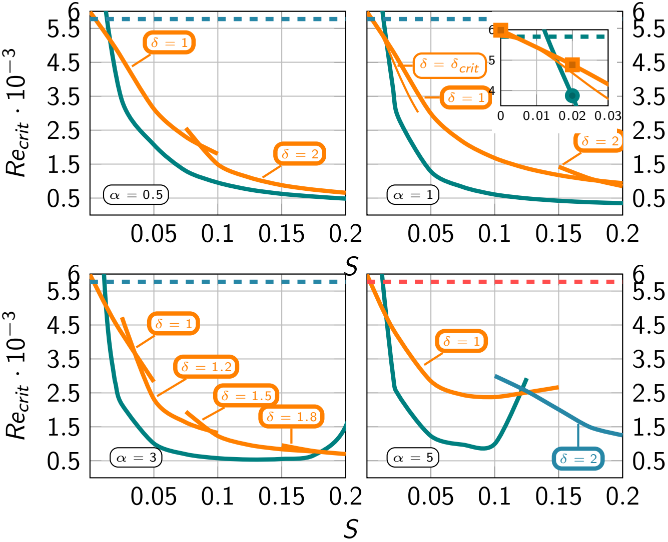


# 3D instability

## 3D stationary vortex

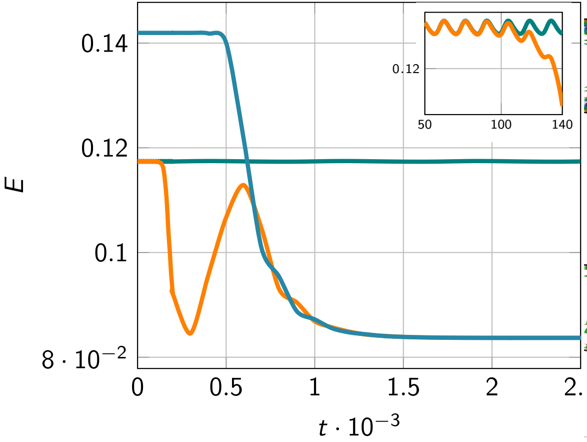


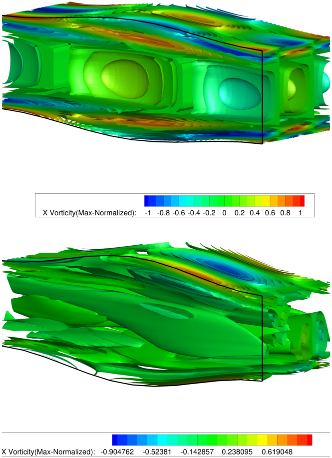
# Competing instabilities



# 3D Saturation

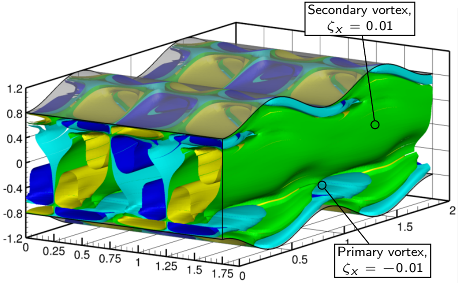
$\alpha = 1, S = 0.2, Re = 1000, \mu = 2.25$

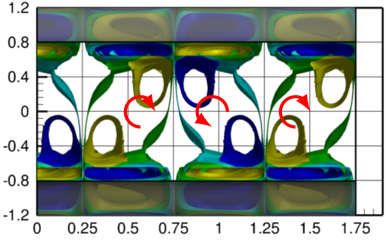


# 3D Saturation

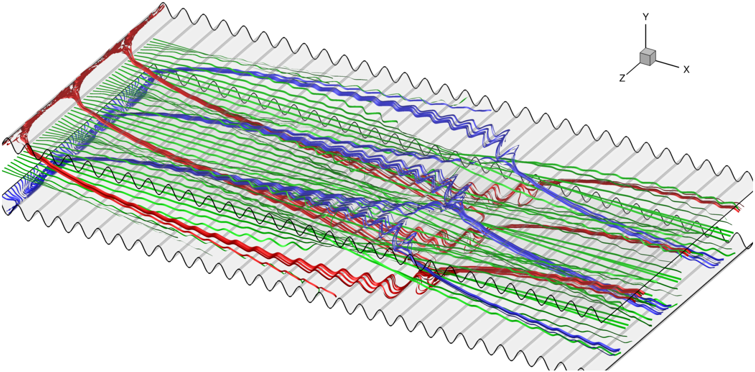
## Flow topology, $\alpha = 1, S = 0.2, Re = 1000, \mu = 2.25$

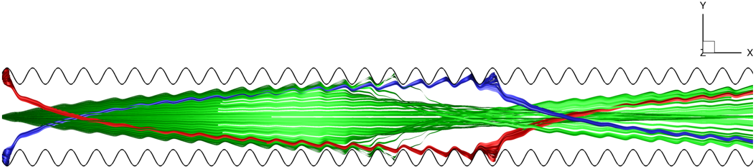


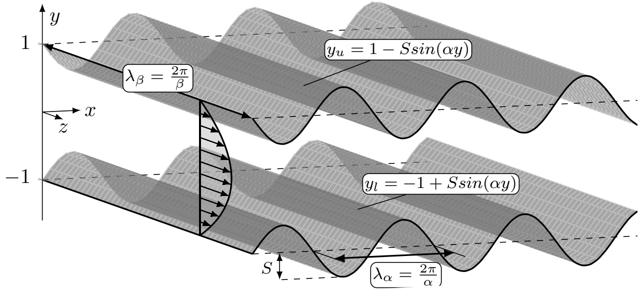
# 3D Saturation

**Flow topology,** $\alpha = 1, S = 0.2, Re = 1000, \mu = 2.25$



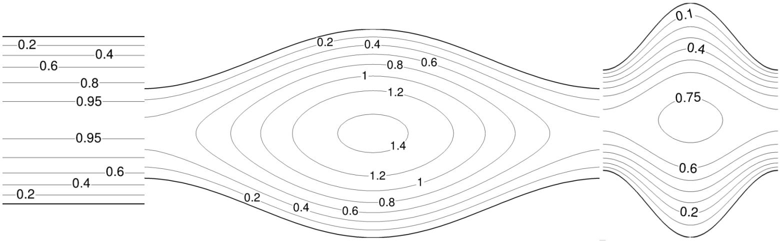
# Transverse corrugation




- $S$ - corrugation amplitude
- $Re = \frac{UL}{\nu}$ - reference flow, $Q_r = \frac{4}{3}$
- $n$ - number of corrugations in computations
- $\alpha$ - spanwise wave number $\rightarrow \lambda_\alpha = \frac{2\pi}{\alpha}$
- $\beta$ - streamwise wave number

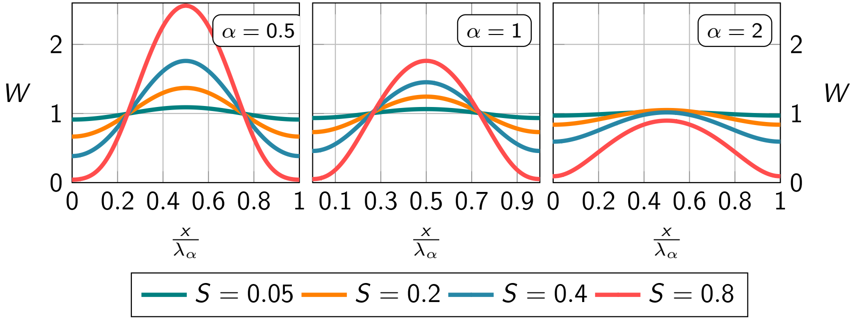
$$(\alpha, S, Re, n, \beta)$$

# 2D base flow


0.2
0.4
0.6
0.8
0.95
0.95
0.6
0.4
0.2
0.2
0.4
0.8
0.6
1
1.2
1.4
1.2
1
0.8
0.2
0.4
0.6
0.1
0.4
0.75
0.6
0.2

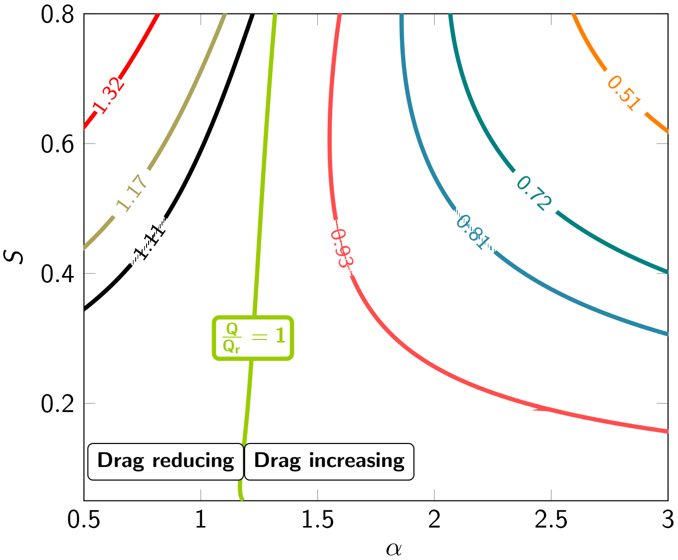
# Base flow

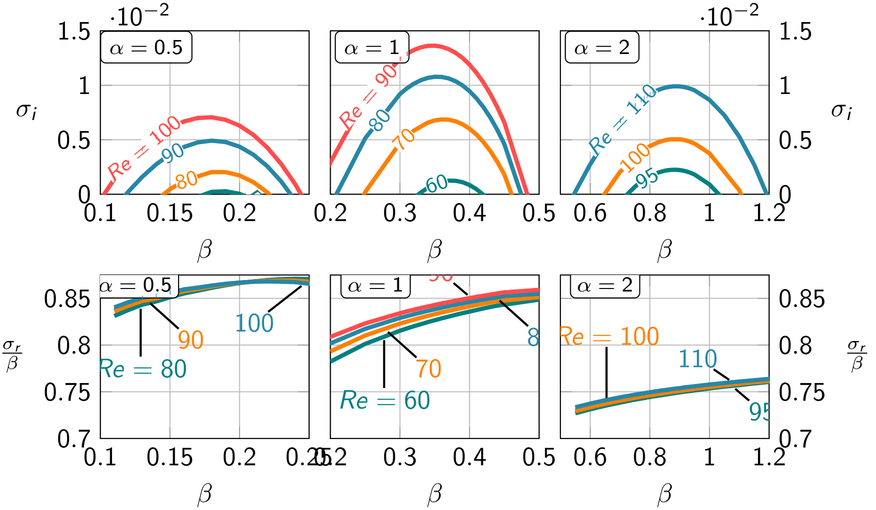
## velocity profile at $y = 0$ plane



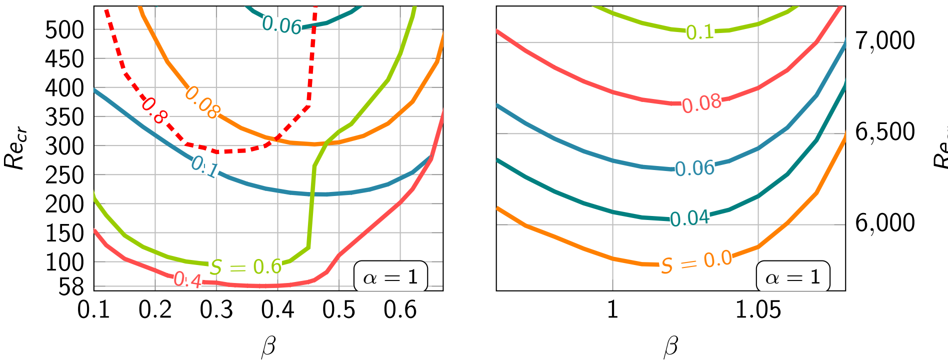
# Base flow

## Drag reduction!




$\cdot 10^{-2}$
$\alpha = 0.5$
$\alpha = 1$
$\alpha = 2$
$\sigma_i$
$Re = 100$
90
80
$Re = 90$
70
60
$Re = 110$
100
95
$\beta$
$\frac{\sigma_r}{\beta}$
$Re = 80$
$Re = 60$
$Re = 100$
110

# Comparison with the TS mode



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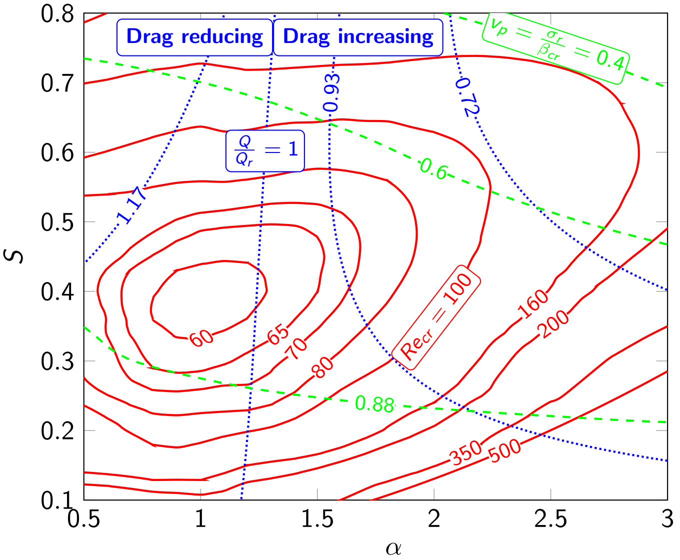
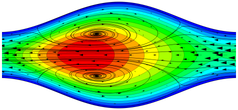
# 3D travelling wave

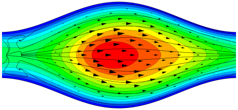


# 3D Saturation

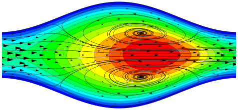
## Flow pattern $\alpha = 1, S = 0.4, Re = 80, \alpha = 1, \beta = 0.4$



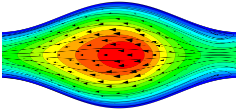
$z = \frac{\lambda_\beta}{4}$



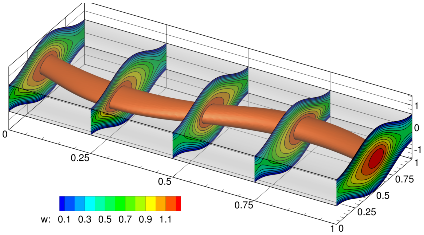
$z = \frac{\lambda_\beta}{2}$

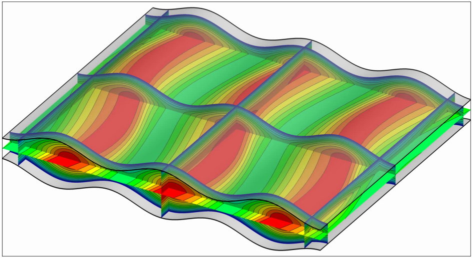


$z = \frac{3\lambda_\beta}{4}$



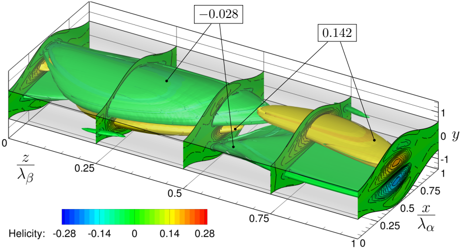
$z = \lambda_\beta$

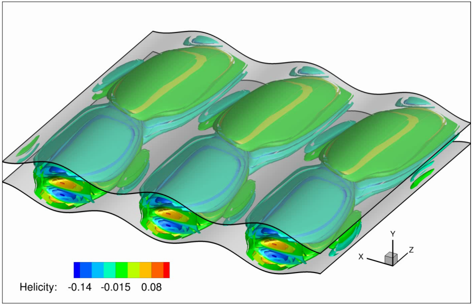


# 3D Saturation

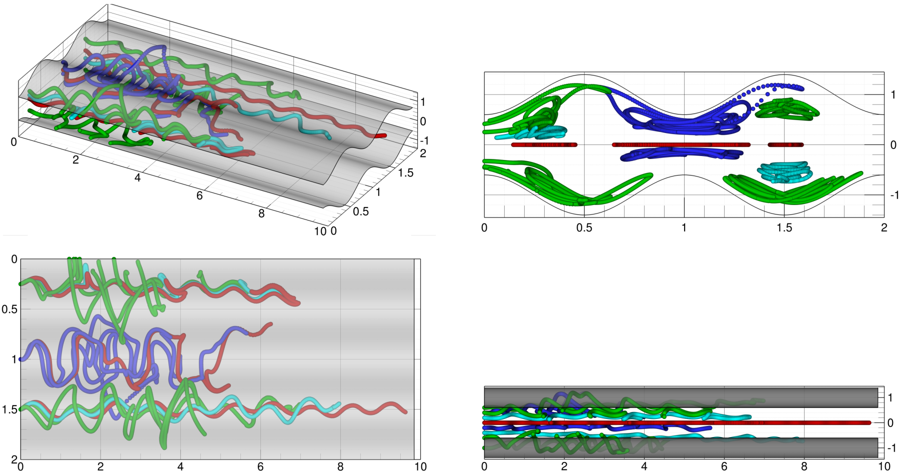
## Helicity



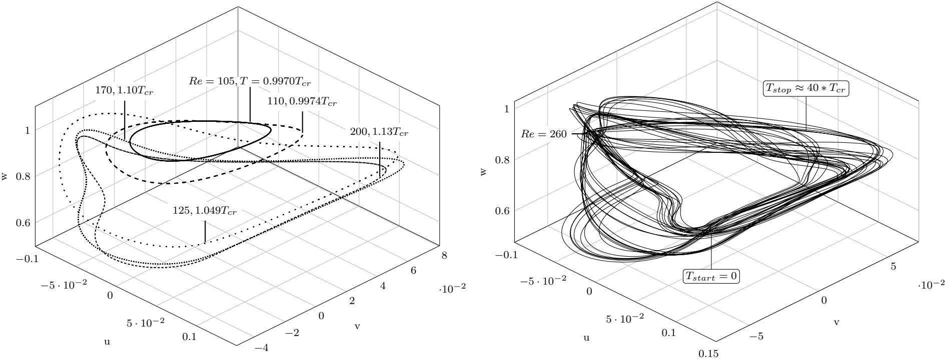



# 3D Saturation

## Streaklines



# Velocity vector phase space trajectories



# Conclusions

- Large scale wall corrugation promotes various types of instabilities
- Low Reynolds number destabilization is possible
- ...