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On implicit LES / under-resolved DNS via spectral element methods

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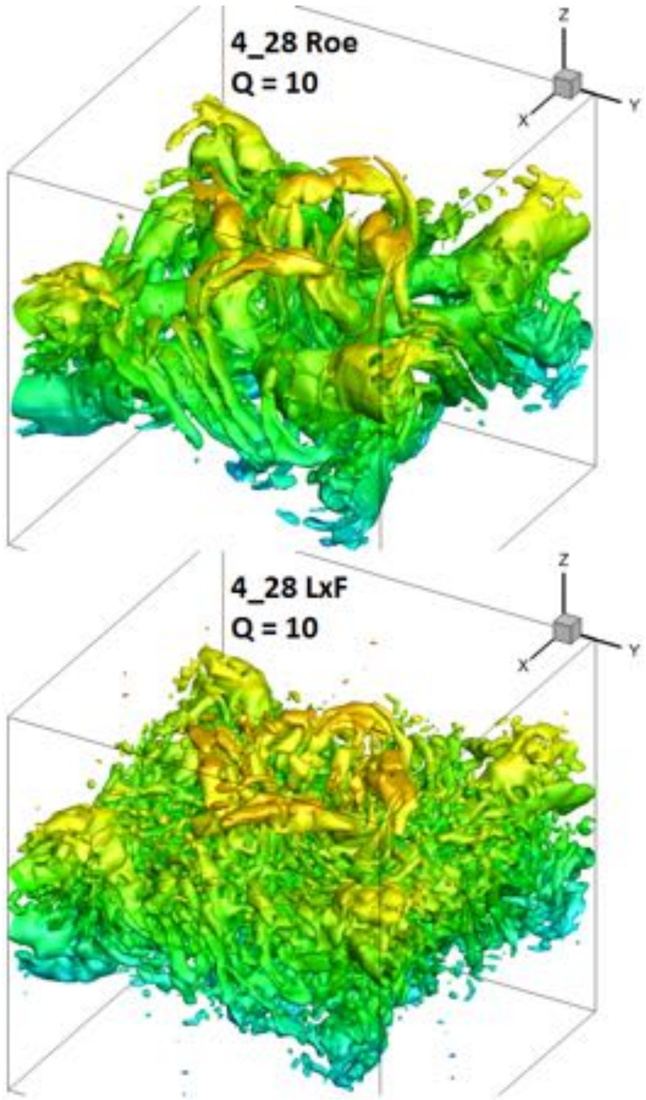
Nektar++ Workshop 2016
June 7, 2016

Introduction & outline

- Implicit LES vs. under-resolved DNS
- Why does it work and how to apply it ?
- Understanding the numerics is essential !
- Eigensolution (dispersion-diffusion) analysis
- Upwind DG vs. CG+SVV
- Numerical experiments with Nektar++
- Focusing on accuracy and robustness



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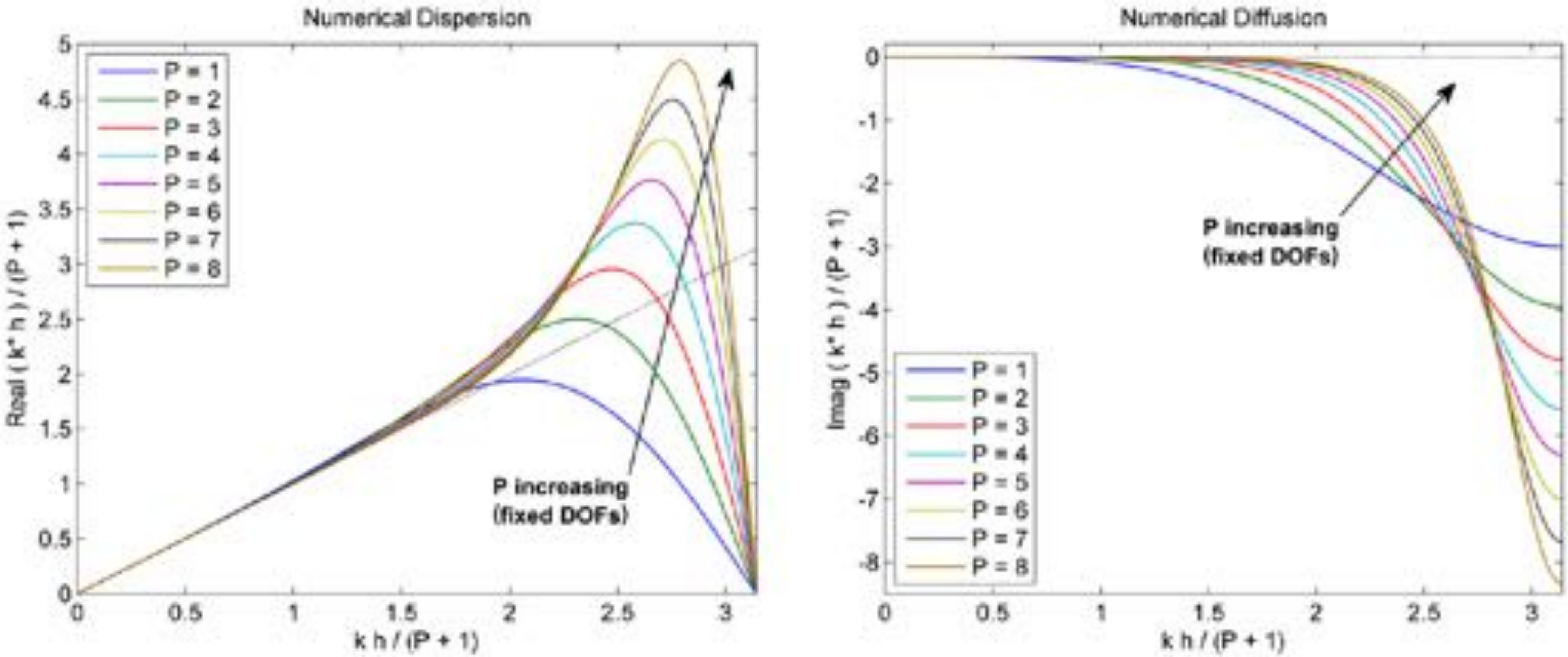
Eigensolution analysis for DG – linear advection in 1D



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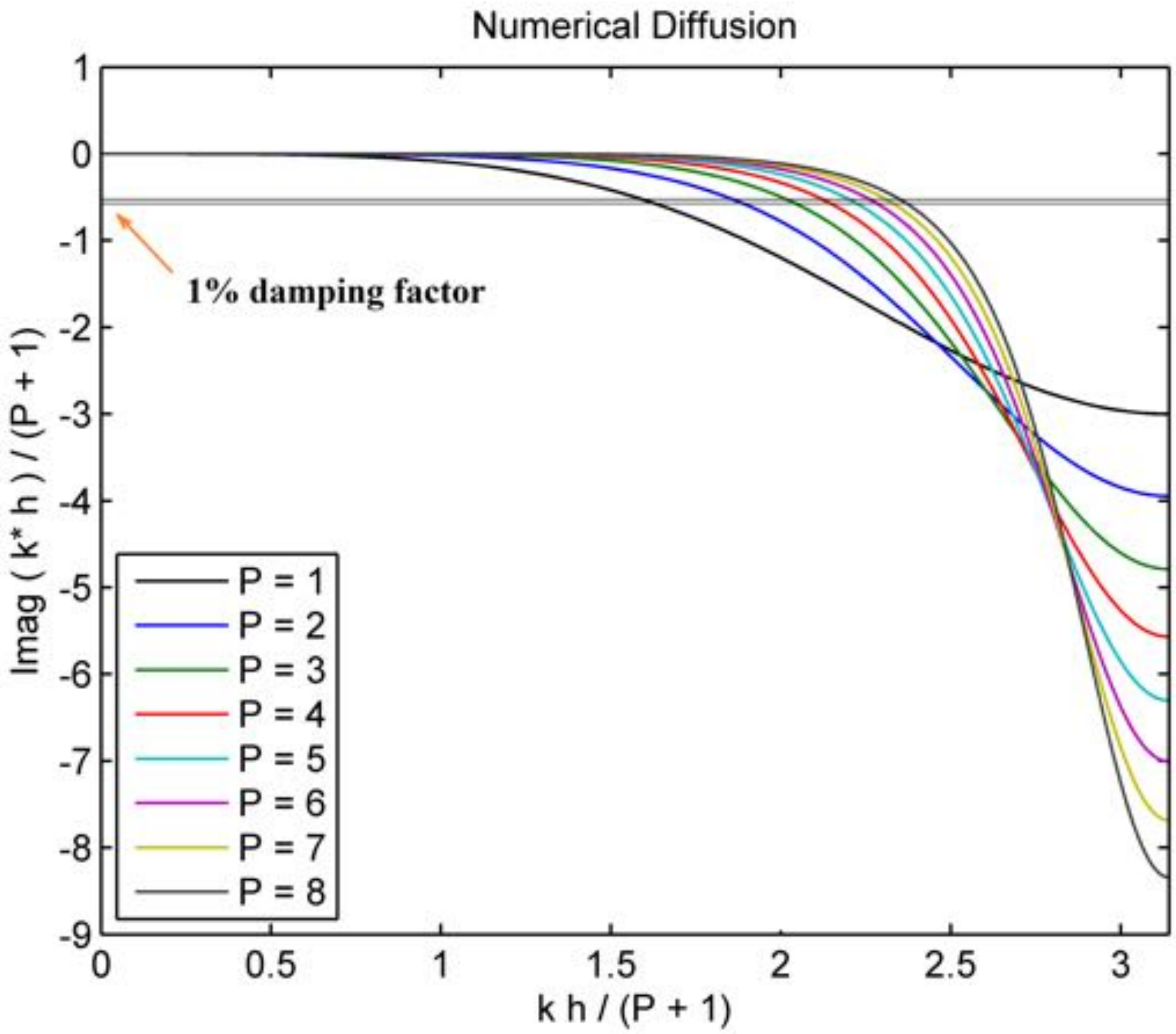
Eigensolution analysis for DG – the 1% rule



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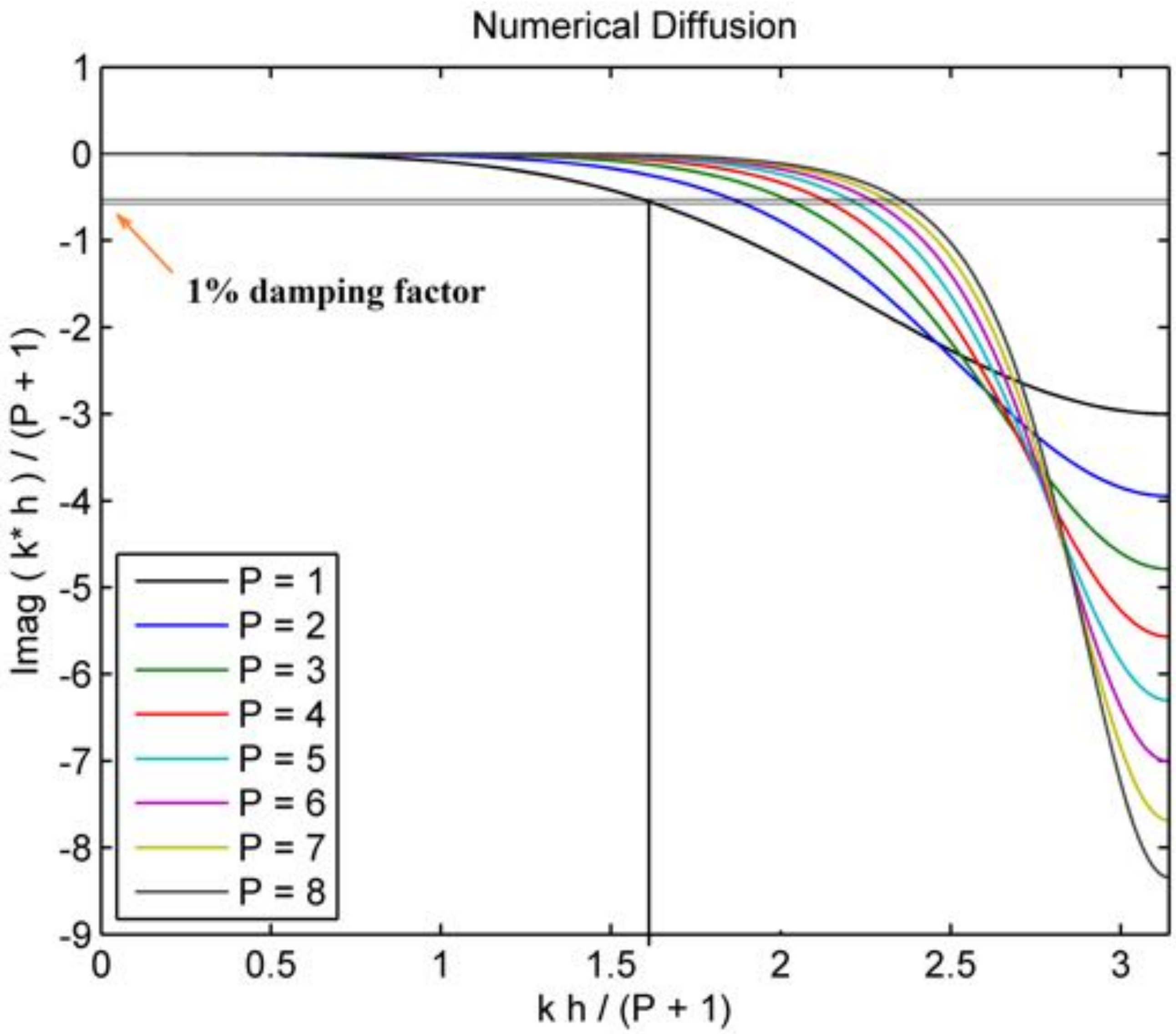

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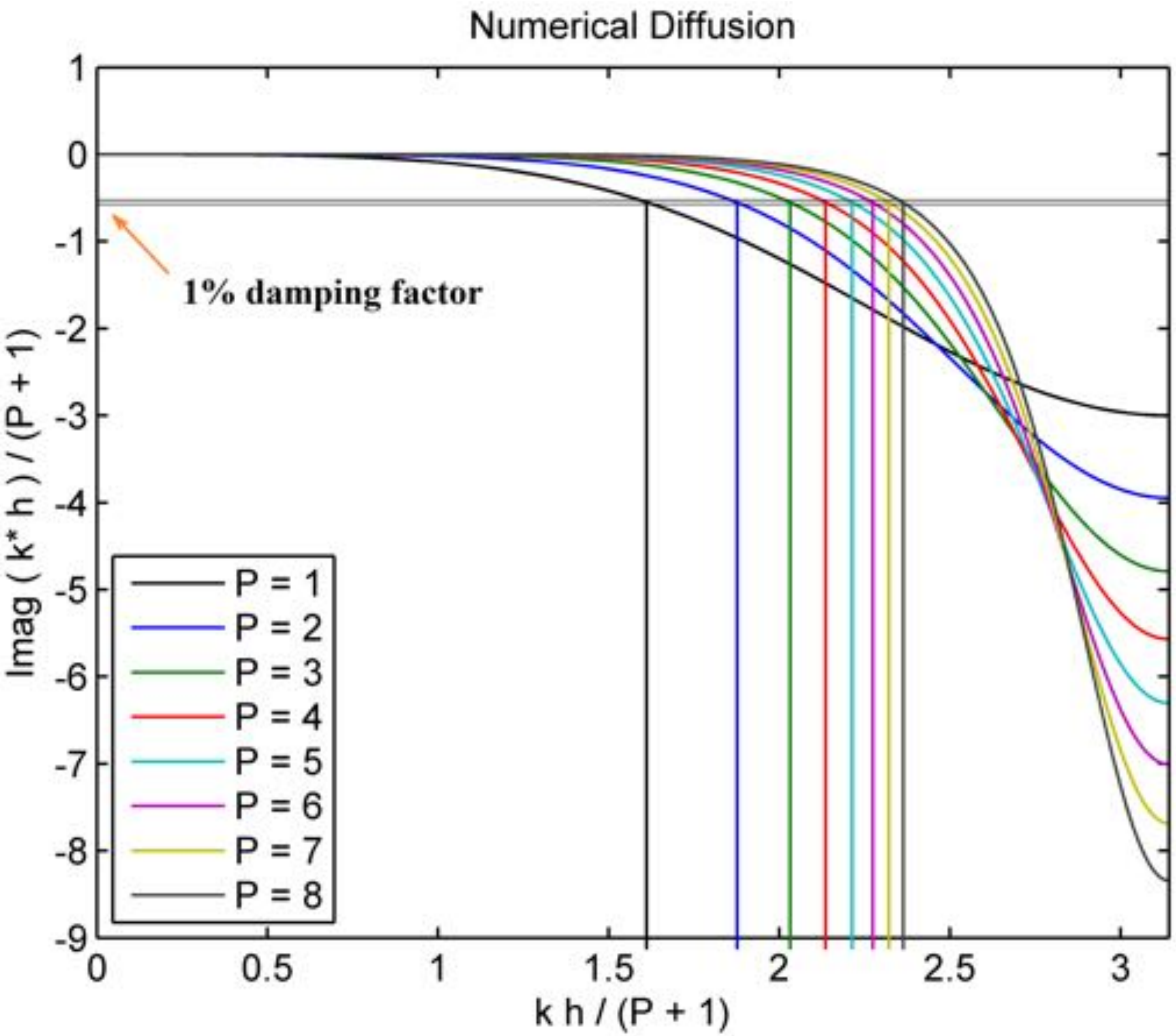

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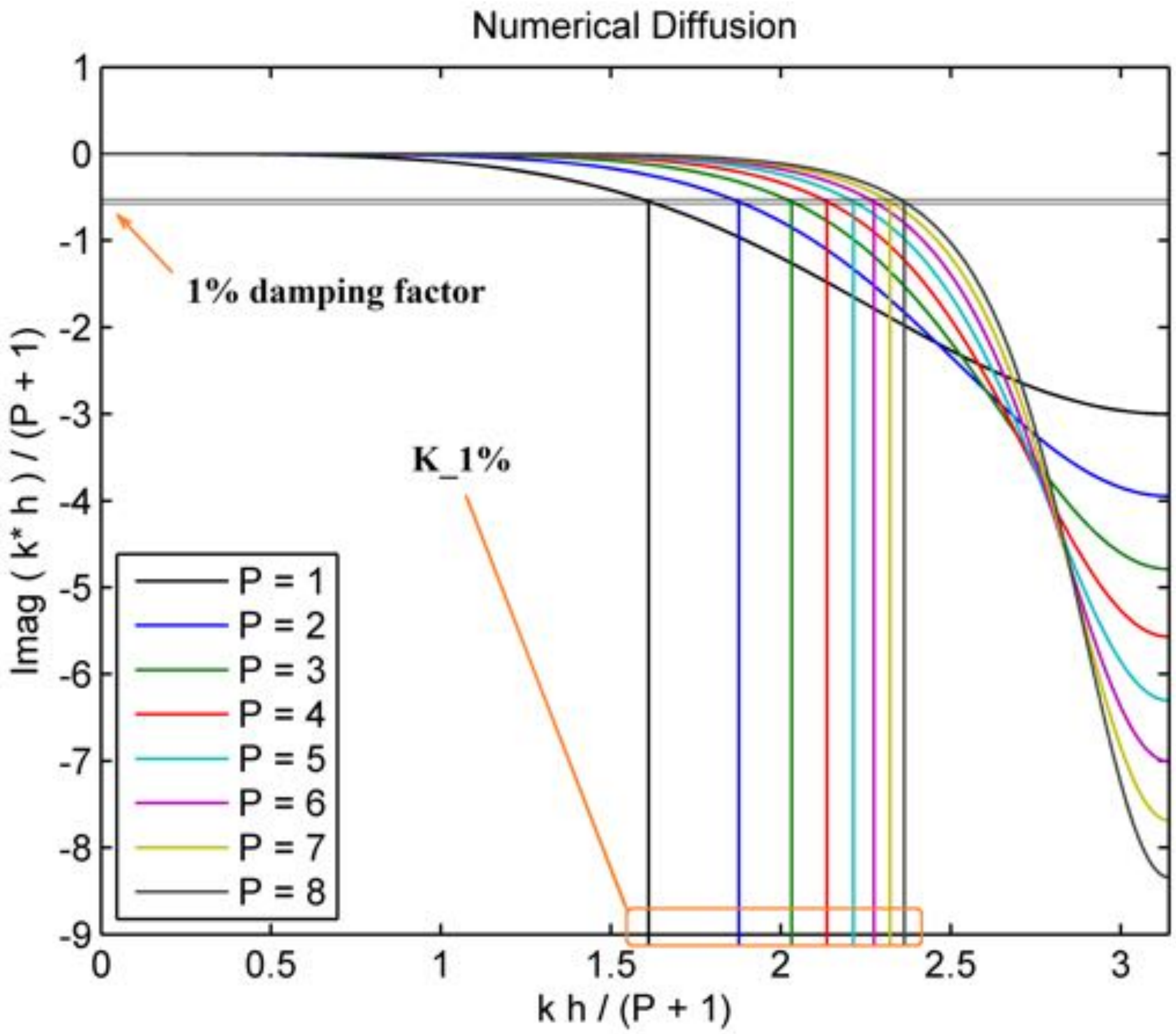
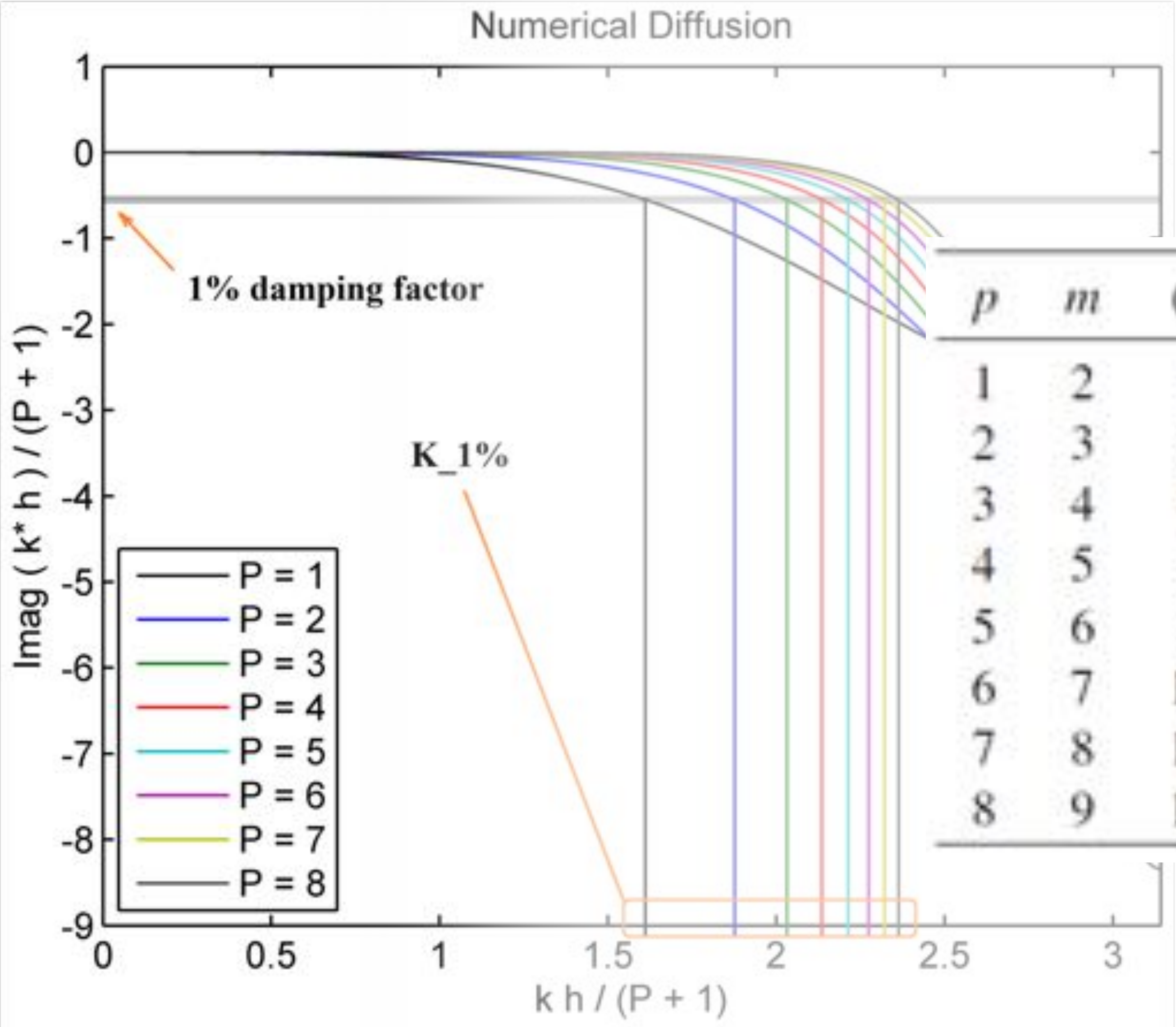
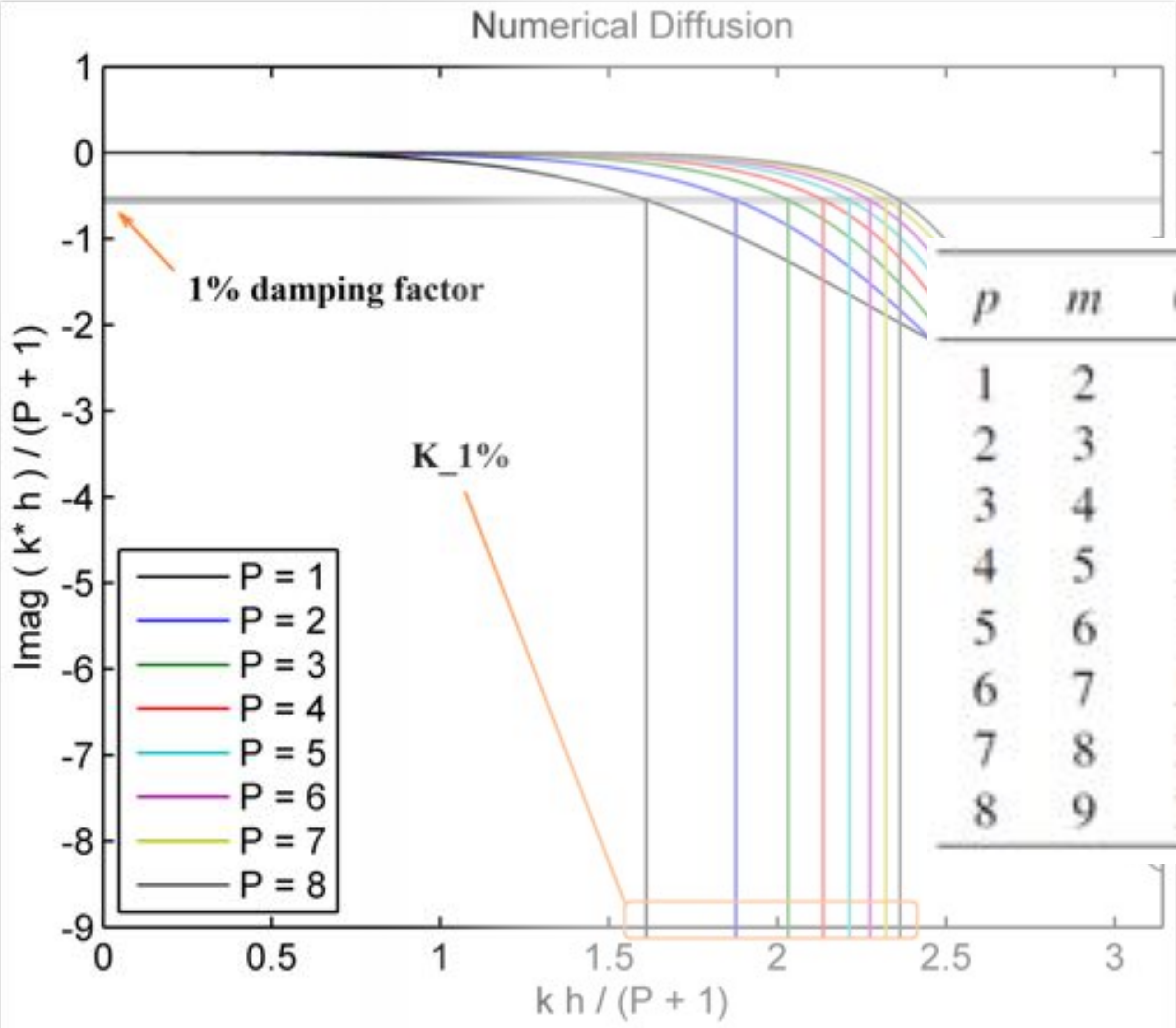


Eigensolution analysis for DG – the 1% rule



p	m	$(kh)^{1d}_{1\%}$	$2\pi/(k\hbar)^{1d}_{1\%}$	$(\Delta/h)^{1d}_{1\%}$
1	2	1.127	11.150	2.788
2	3	2.616	7.205	1.201
3	4	4.330	5.804	0.726
4	5	6.164	5.097	0.510
5	6	8.071	4.671	0.389
6	7	10.027	4.386	0.313
7	8	12.018	4.183	0.261
8	9	14.035	4.029	0.224

Eigensolution analysis for DG – the 1% rule



LINEAR ADVECTION

p	m	$(kh)^{1d}_{1\%}$	$2\pi/(k\hbar)^{1d}_{1\%}$	$(\Delta/h)^{1d}_{1\%}$
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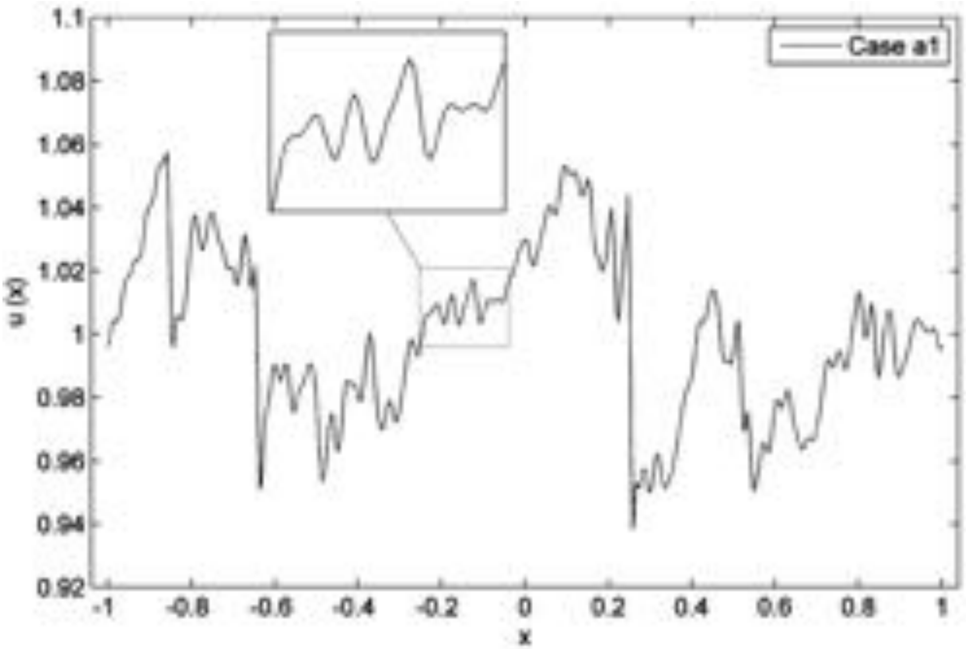
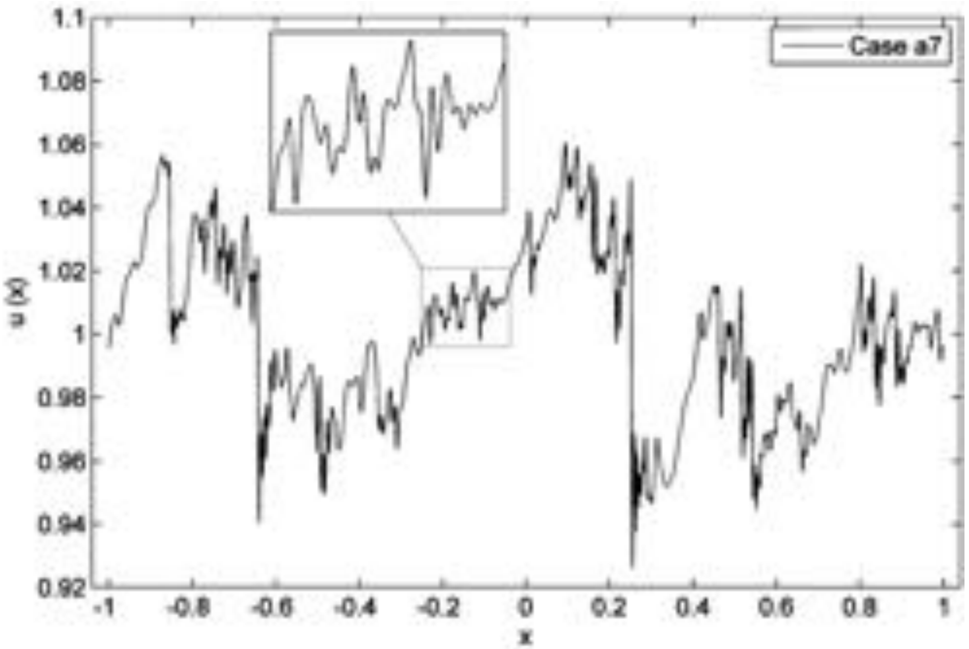
(std. upwinding)

Eigensolution analysis for DG – tests in Burgers turbul.

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial x} = \frac{A_F}{\sqrt{\Delta t}} \sum_{N \in \mathbb{N}_F} \frac{\sigma_N(t)}{\sqrt{|N|}} \exp\left(i \frac{2\pi N}{L} x \right)$$



$p = 1, n_{el} = 2048$



$p = 7, n_{el} = 512$

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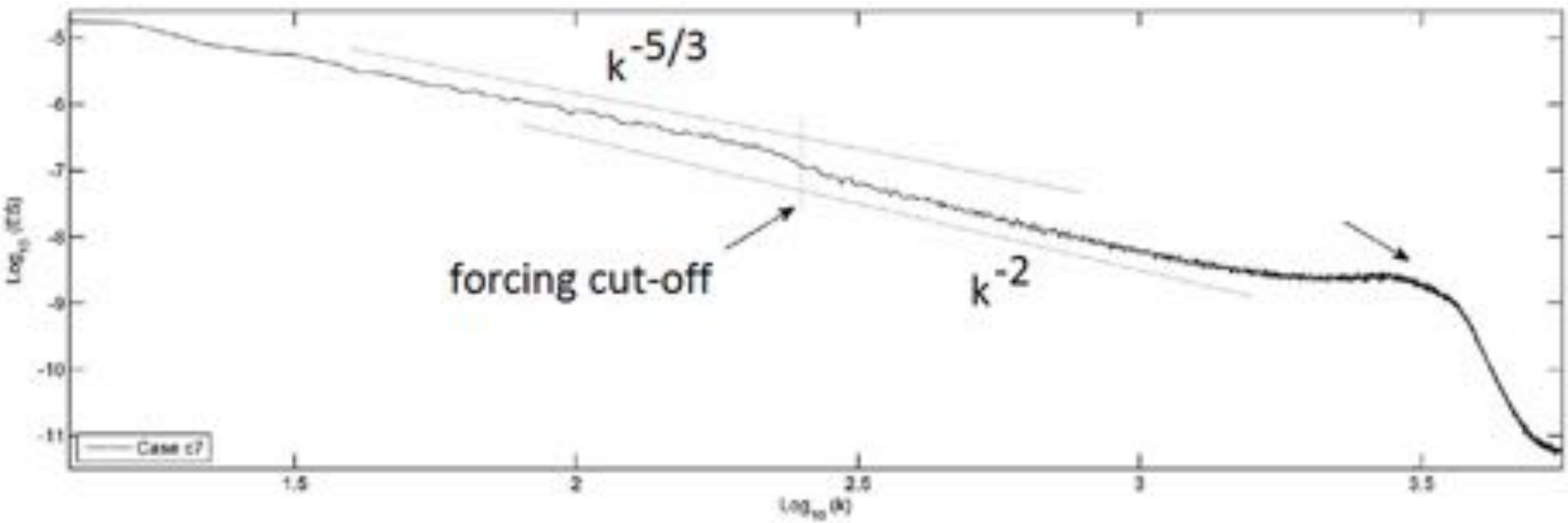
Eigensolution analysis for DG – tests in Burgers turbul.

p	m	$(kh)^{1d}_{1\%}$	$2\pi/(kh)^{1d}_{1\%}$	$(\Delta/h)^{1d}_{1\%}$
⋮				
6	7	10.027	4.386	0.313
7	8	12.018	4.183	0.261
8	9	14.035	4.029	0.224

$$k_{1\%} = \frac{(kh)^{1d}_{1\%}}{h} = \frac{12.018}{2/512} \approx 3077$$

$$\Longrightarrow \log_{10} k_{1\%} \approx 3.49$$



Eigensolution analysis for DG – tests in Burgers turbul.

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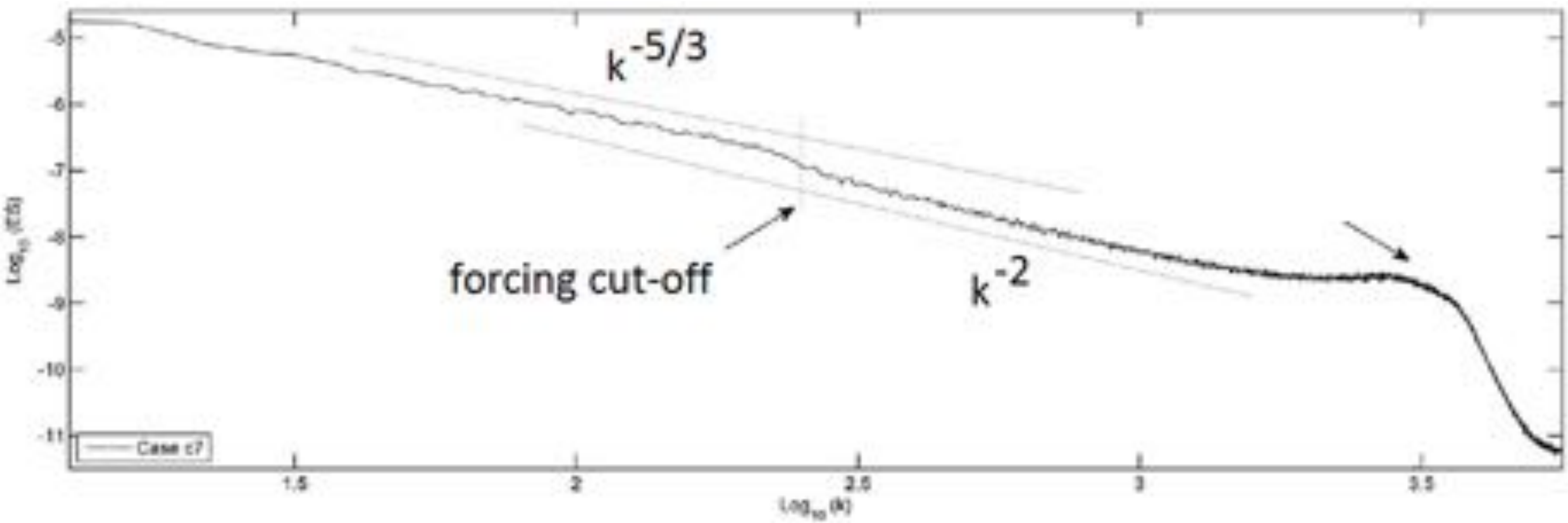

Eigensolution analysis for DG – tests in Burgers turbul.

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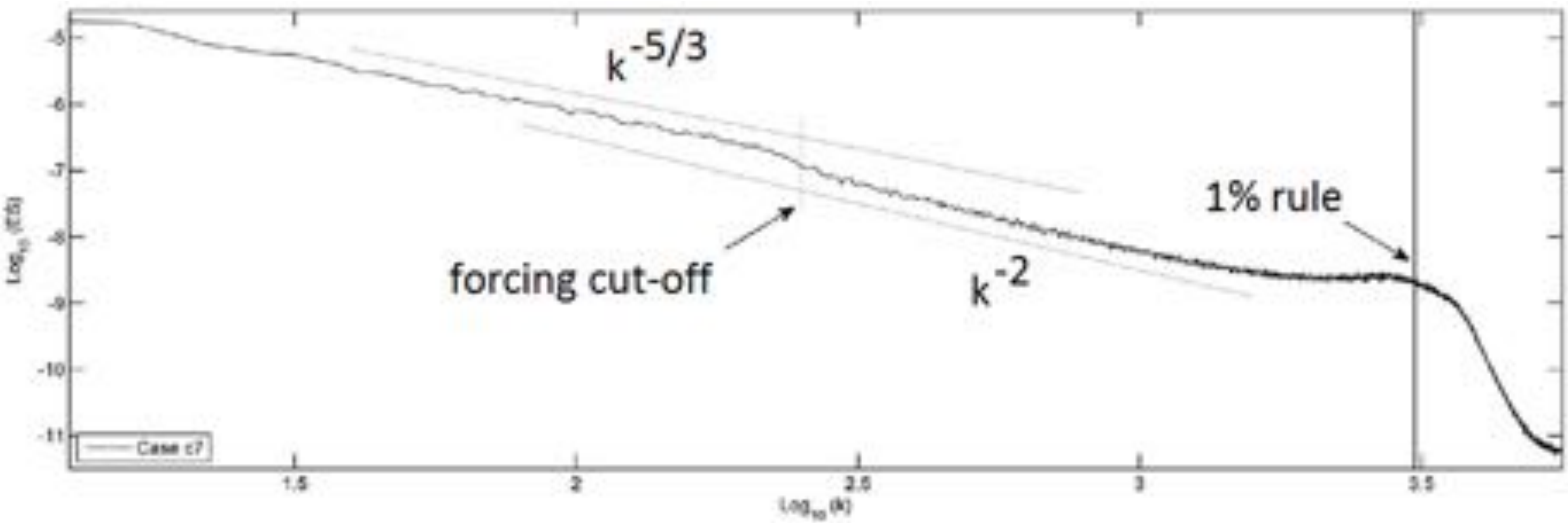
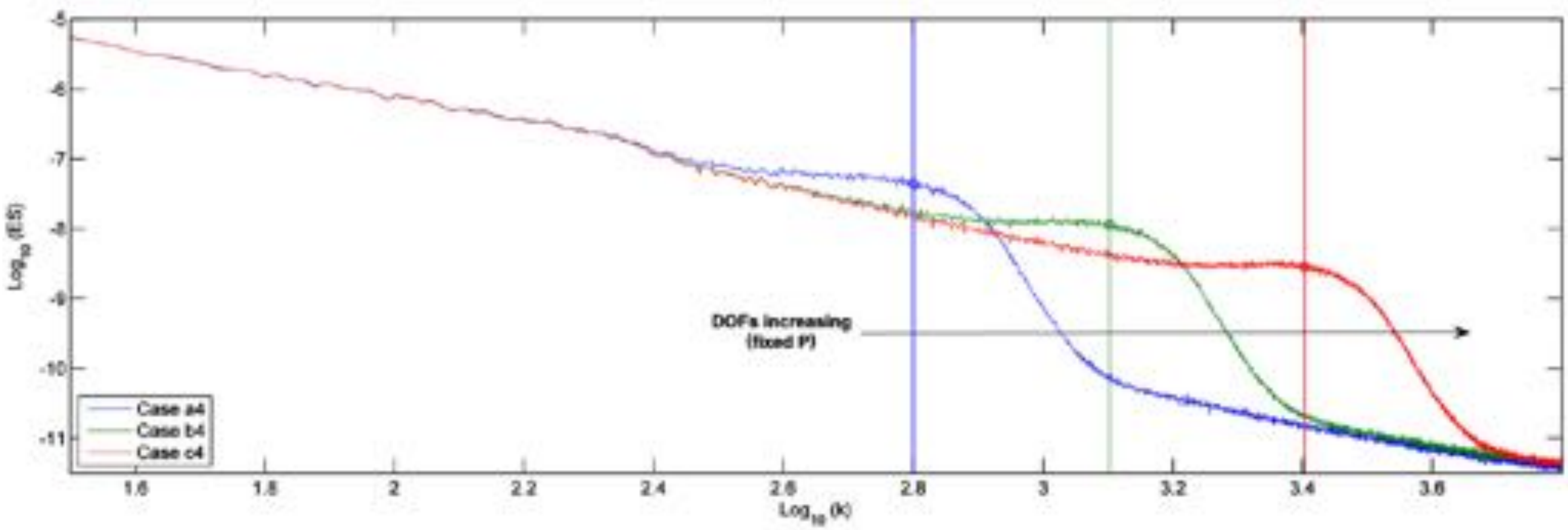
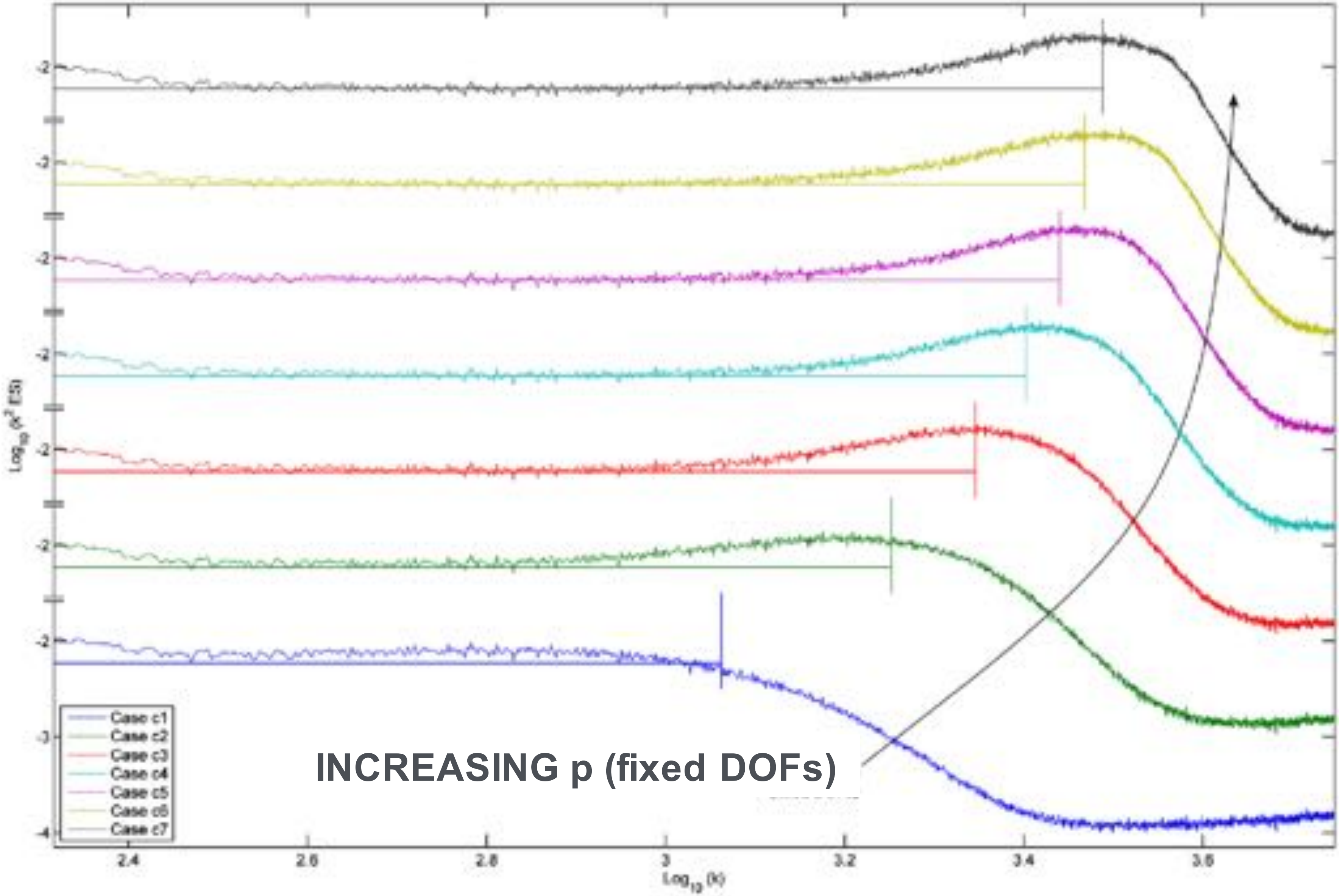


Eigensolution analysis for DG – tests in Burgers turbul.

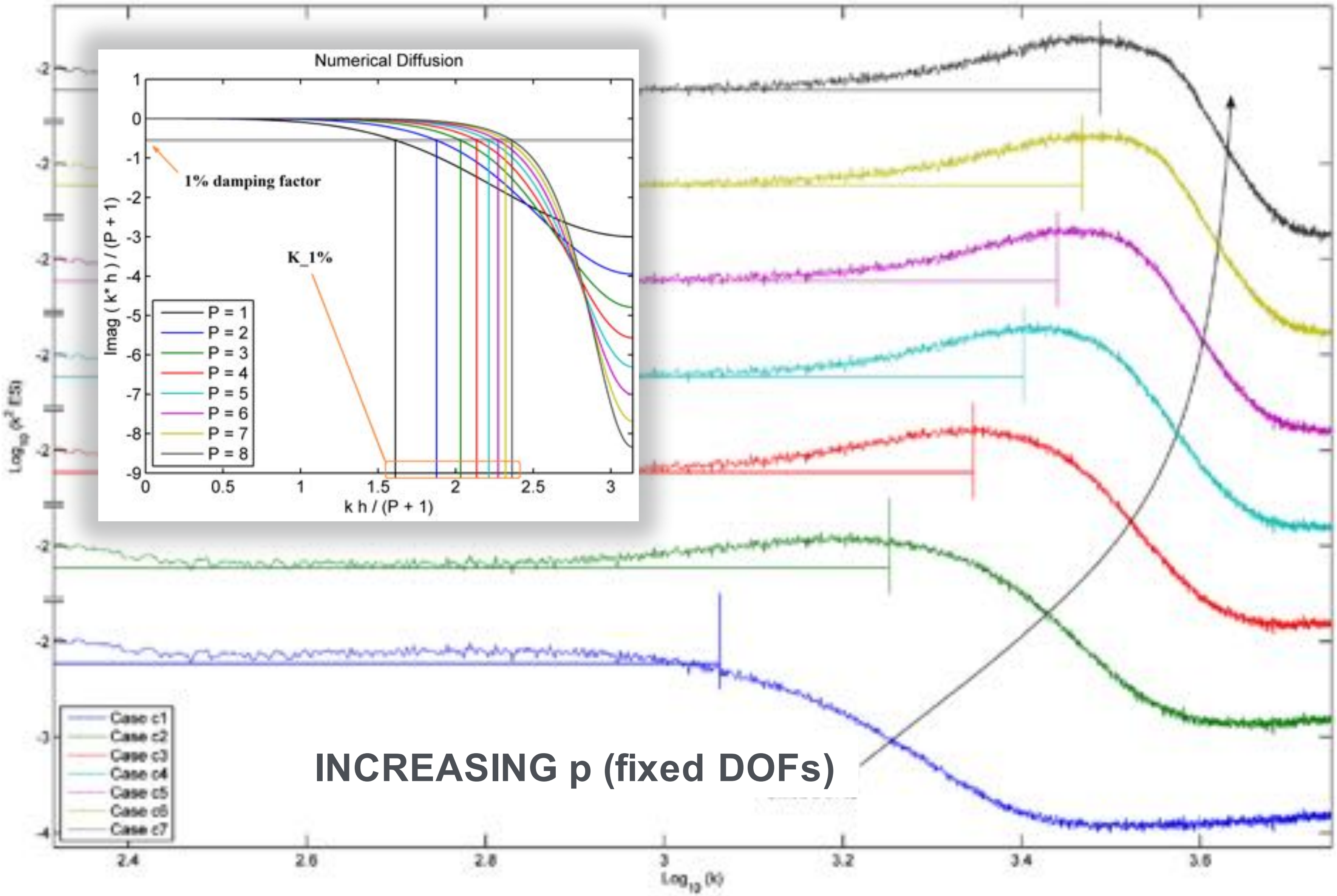
MESH REFINEMENT (p = 4)



Eigensolution analysis for DG – tests in Burgers turbul.



Eigensolution analysis for DG – tests in Burgers turbul.



Eigensolution analysis for CG – insights into SVV

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = \mu \frac{\partial}{\partial x}\left(\mathcal{Q} \star \frac{\partial u}{\partial x}\right) \approx -\mu \sum_k k^2\, \hat{\mathcal{Q}}_k\, \hat{u}_k \,\exp(ikx)$$

Eigensolution analysis for CG – insights into SVV

spectral vanishing viscosity

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Eigensolution analysis for CG – insights into SVV

spectral vanishing viscosity

strictly true for spectral methods

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Eigensolution analysis for CG – insights into SVV

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KERNEL ENTRIES NORMALLY INCREASE FROM ZERO

REGULAR DIFFUSION RECOVERED WHEN $\mathcal{Q}_k = 1$ **for all** k

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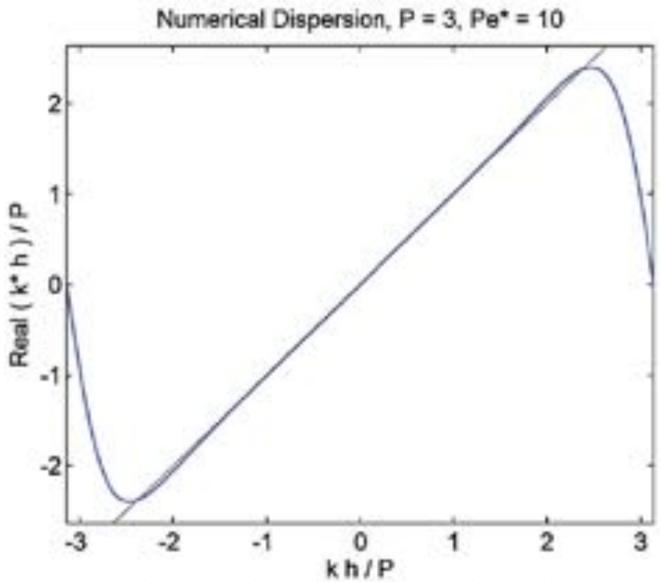
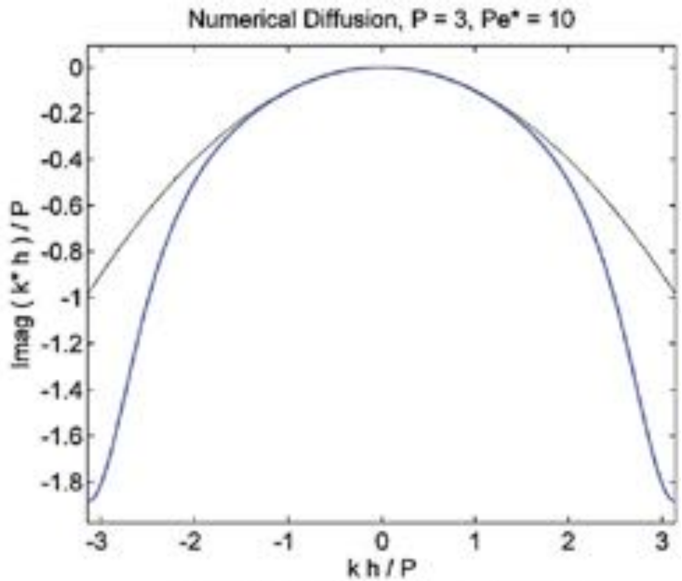
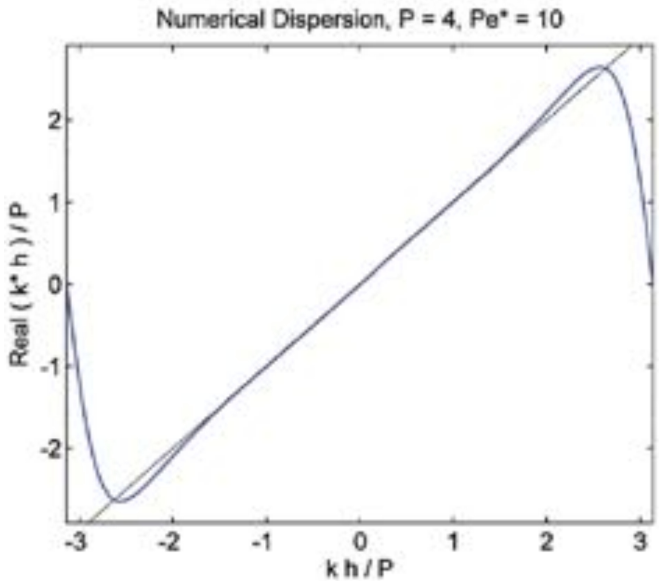
Eigensolution analysis for CG – advection+diffusion

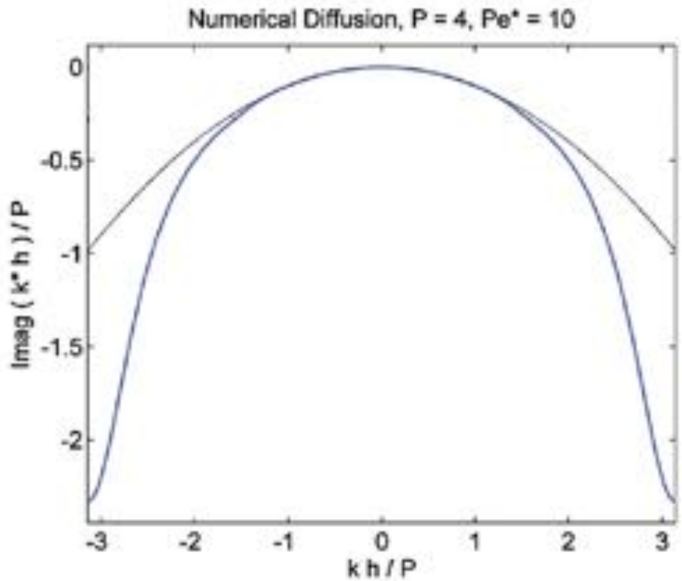




p = 3

$$\mathrm{Pe}^* = a\hbar/\mu = 10$$

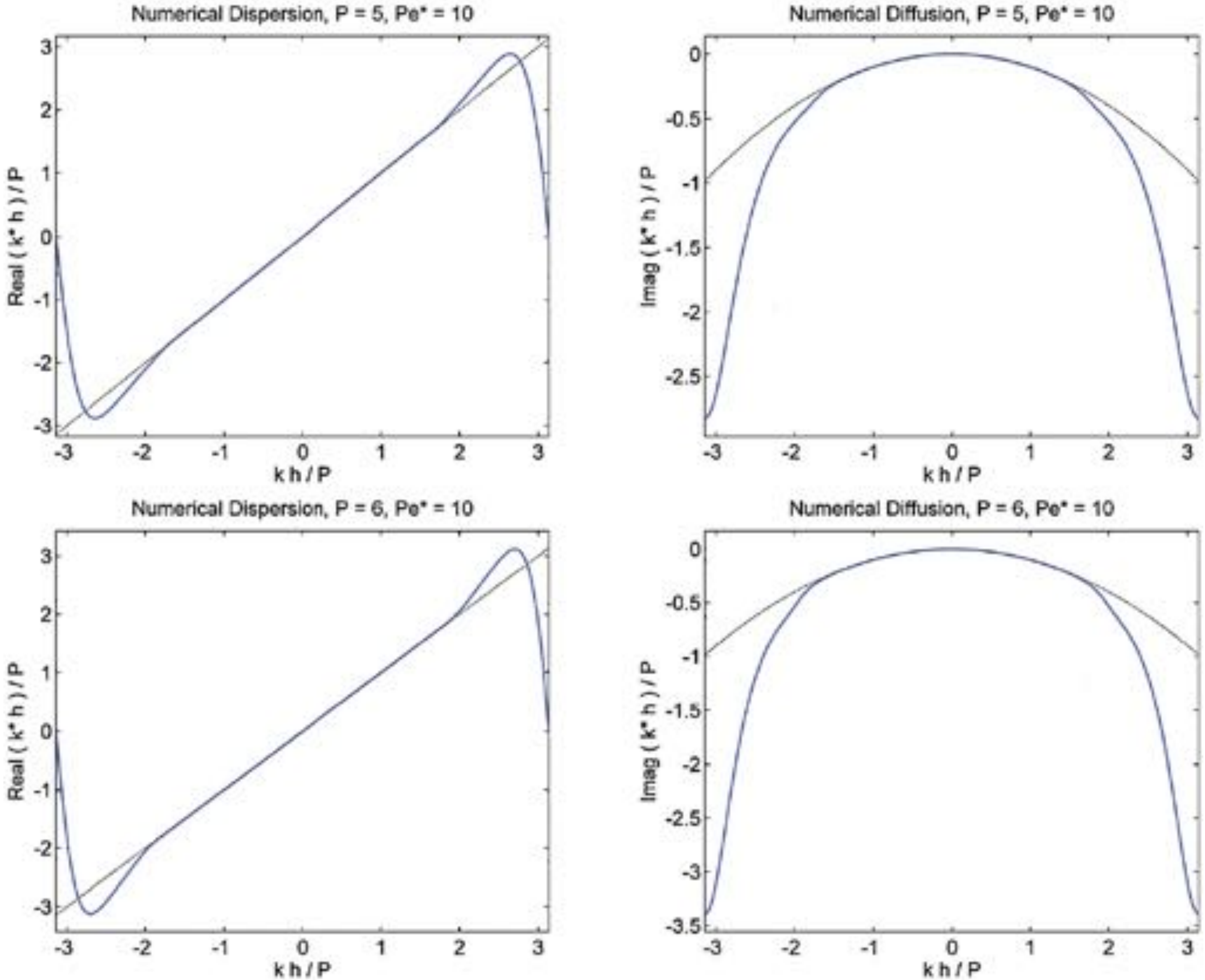




p = 4

$$\mathrm{Pe}^* = a\hbar/\mu = 10$$

Eigensolution analysis for CG – advection+diffusion



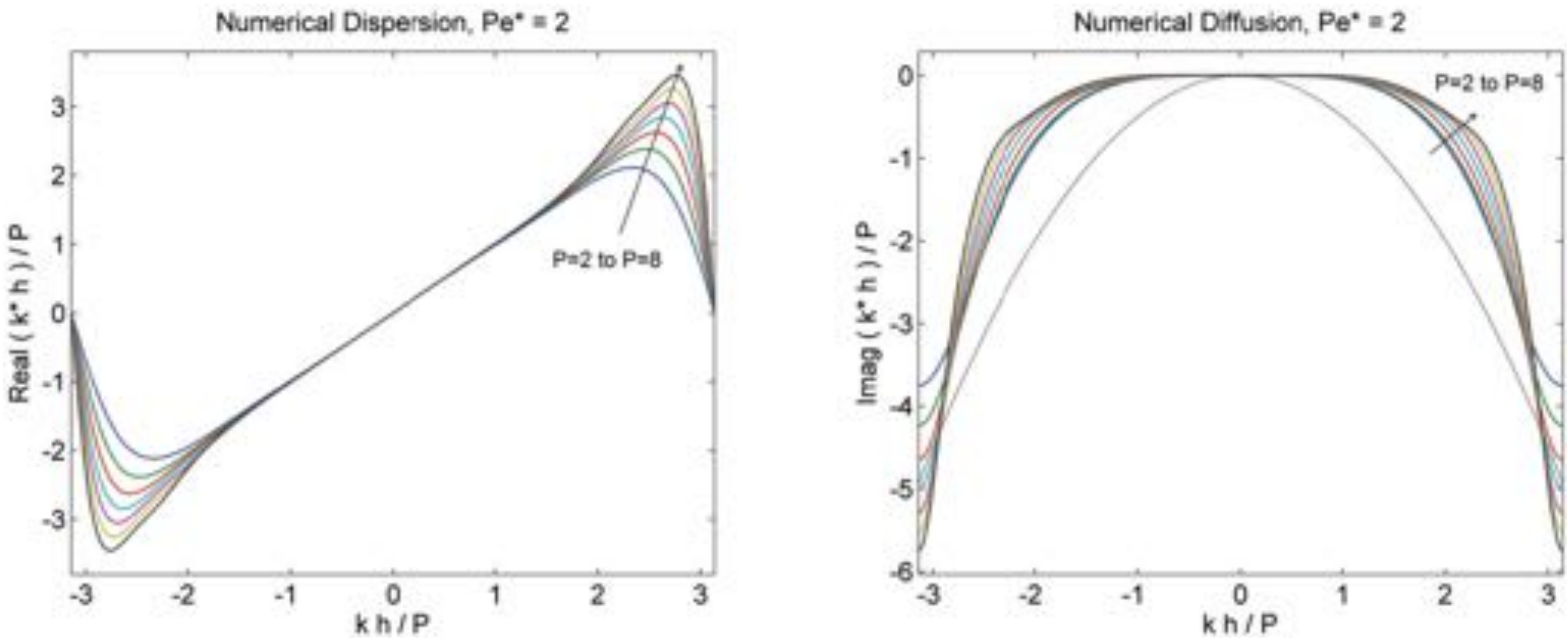
p = 5

$$\mathrm{Pe}^* = a\hbar/\mu = 10$$

p = 6

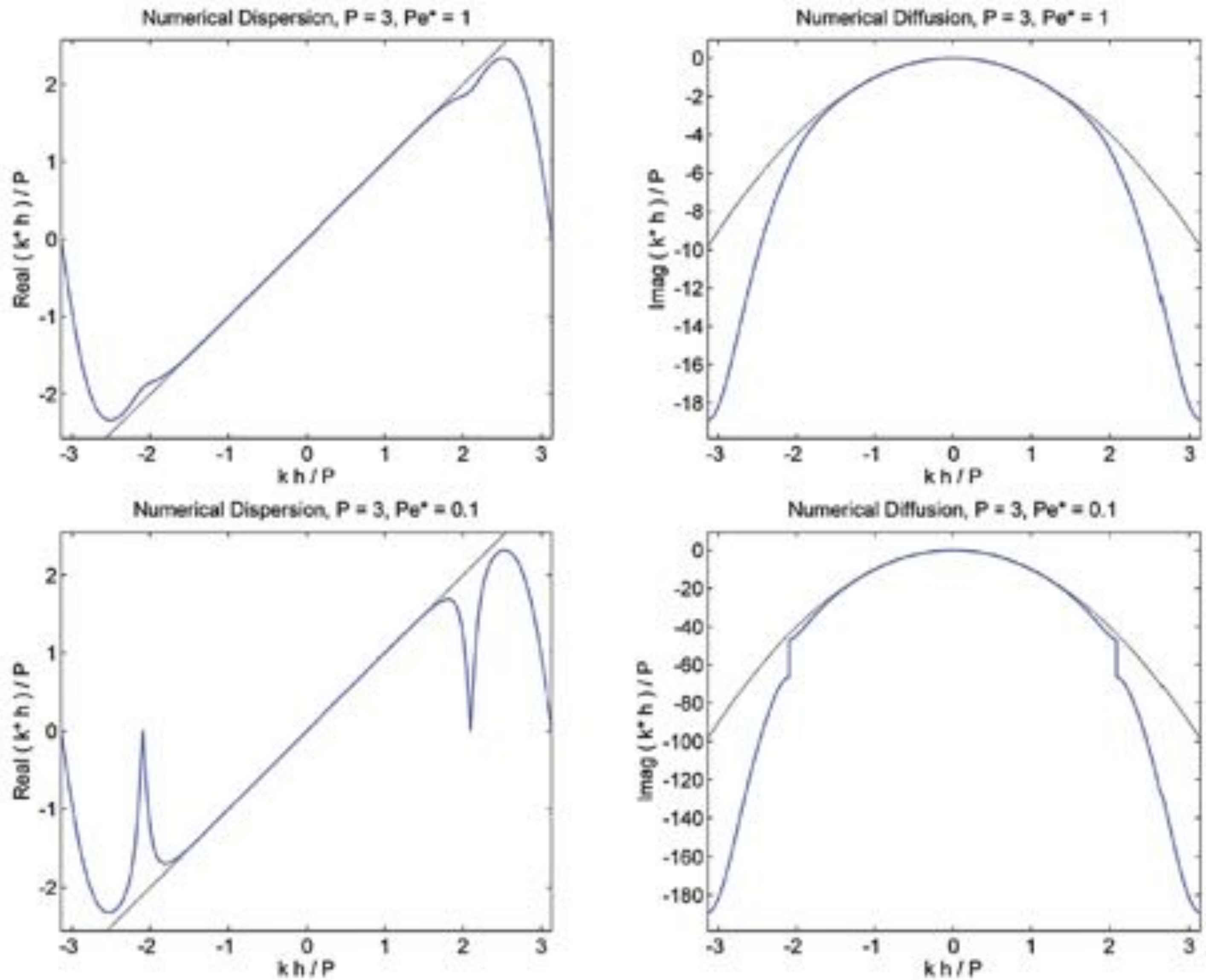
$$\mathrm{Pe}^* = a\hbar/\mu = 10$$

Eigensolution analysis for CG – advection+SVV



$$\text{Proposed 'power kernel'} : \frac{\mathcal{Q}_k}{p} = \left(\frac{k}{p}\right)^{p_{svv}}, \; p_{svv} = p/2$$

Eigensolution analysis for CG – irregular features



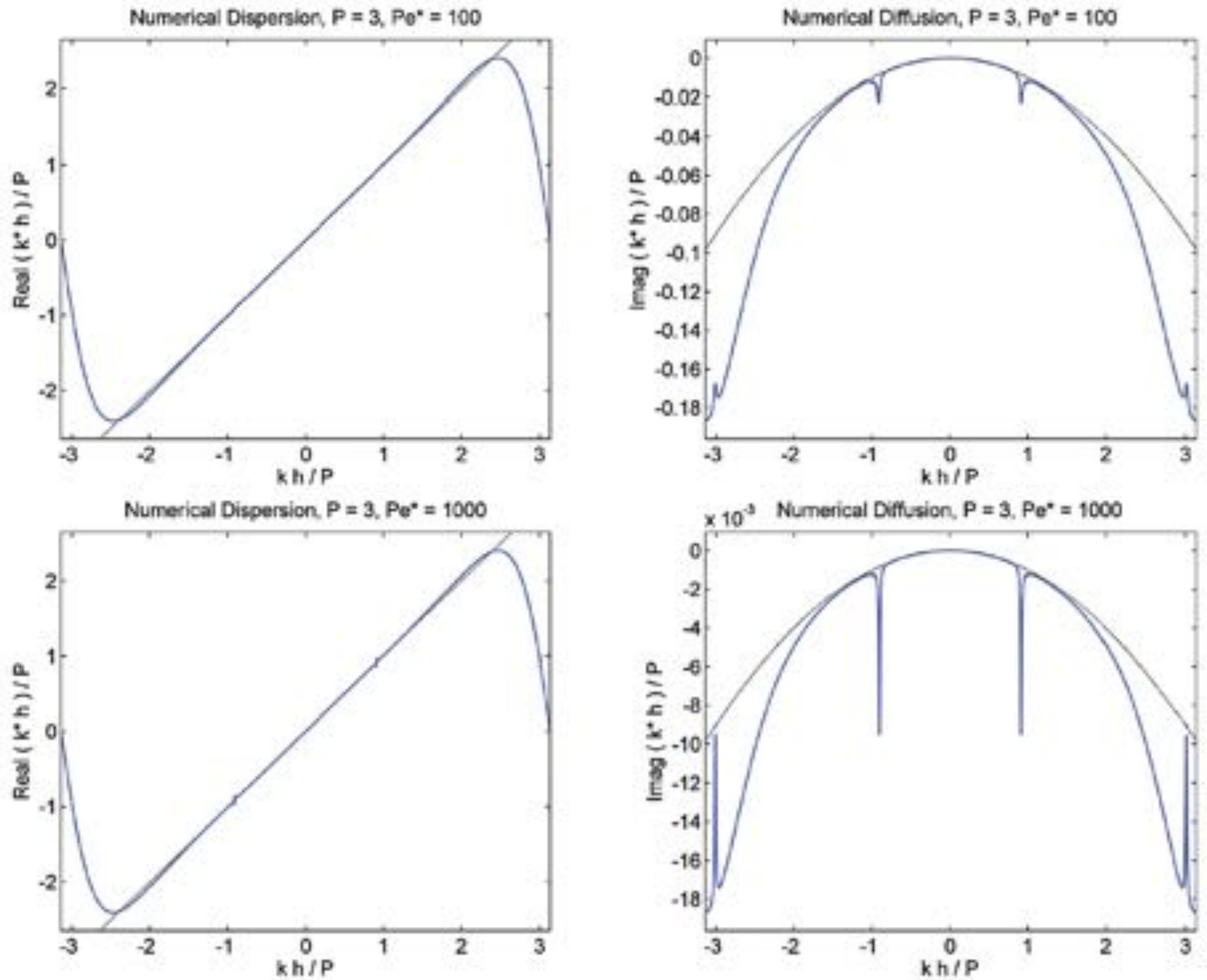
p = 3

$$\mathrm{Pe}^* = a\hbar/\mu = 1$$

p = 3

$$\mathrm{Pe}^* = a\hbar/\mu = 0.1$$

Eigensolution analysis for CG – irregular features



p = 3

$$\mathrm{Pe}^* = a\hbar/\mu = 100$$

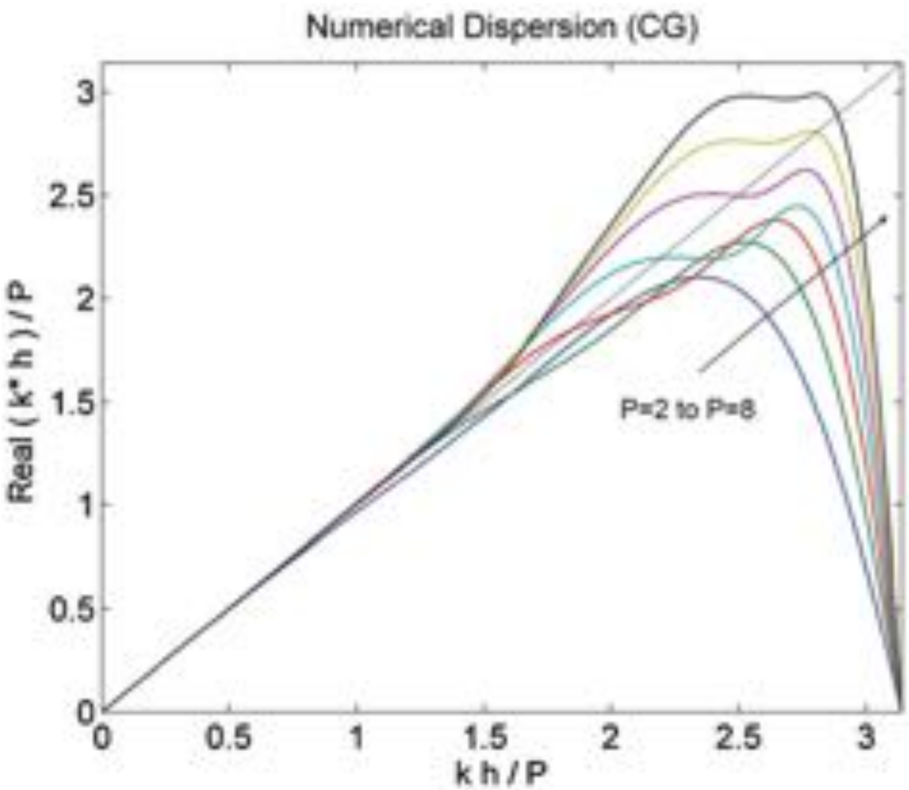
p = 3

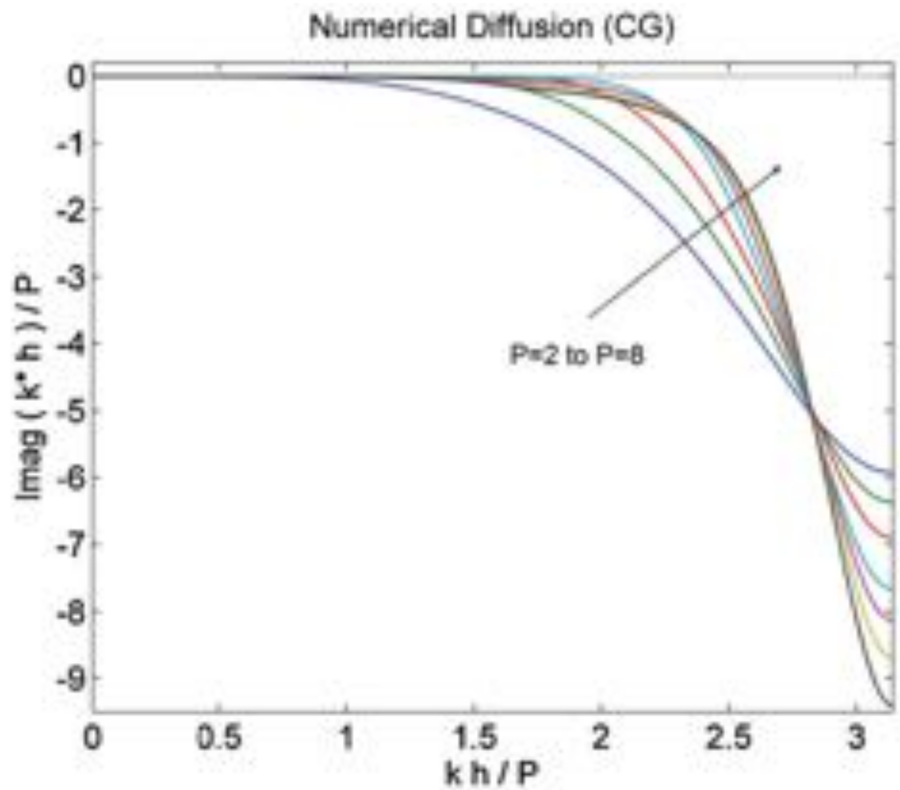
$$\mathrm{Pe}^* = a\hbar/\mu = 1000$$

Eigensolution analysis for CG – a Péclet-free SVV

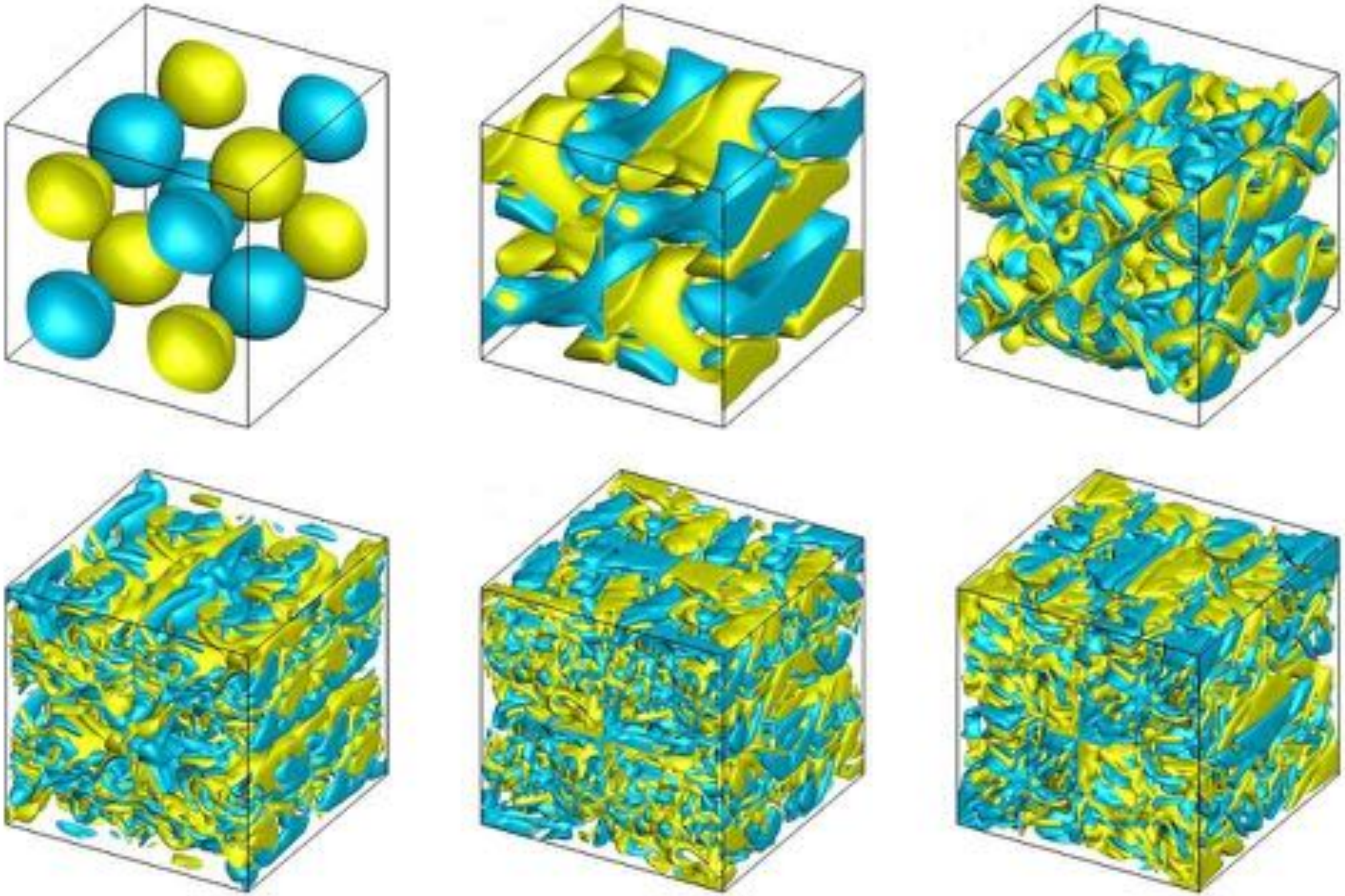
$$\text{Using} \quad \mu \propto \frac{ah}{p} \quad \Longrightarrow \quad \text{fixed} \;\; \mathrm{Pe}^* = a\hbar/\mu$$

(optimized SVV kernel to mimic DG)





Numerical experiments with Nektar++ (inviscid TGV)



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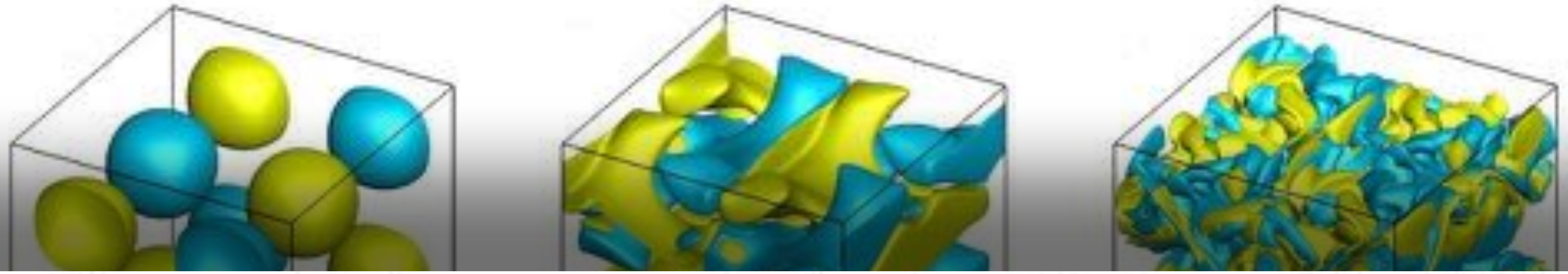
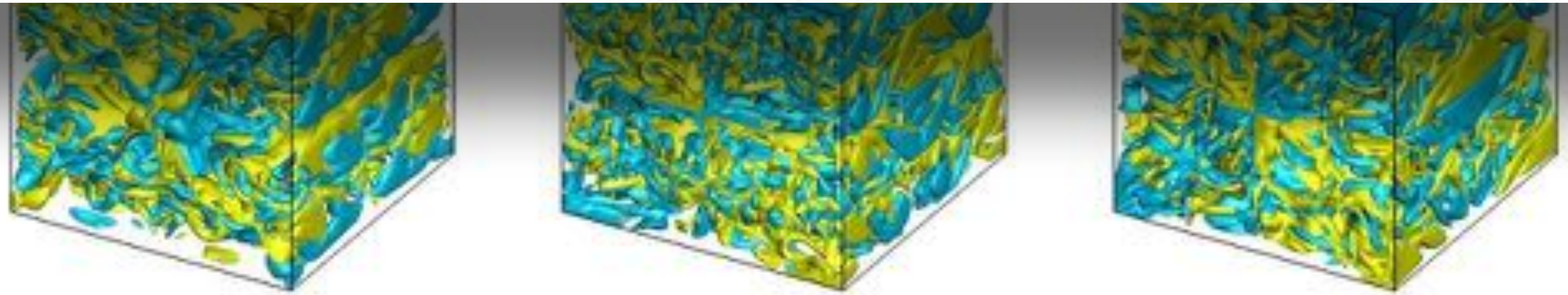
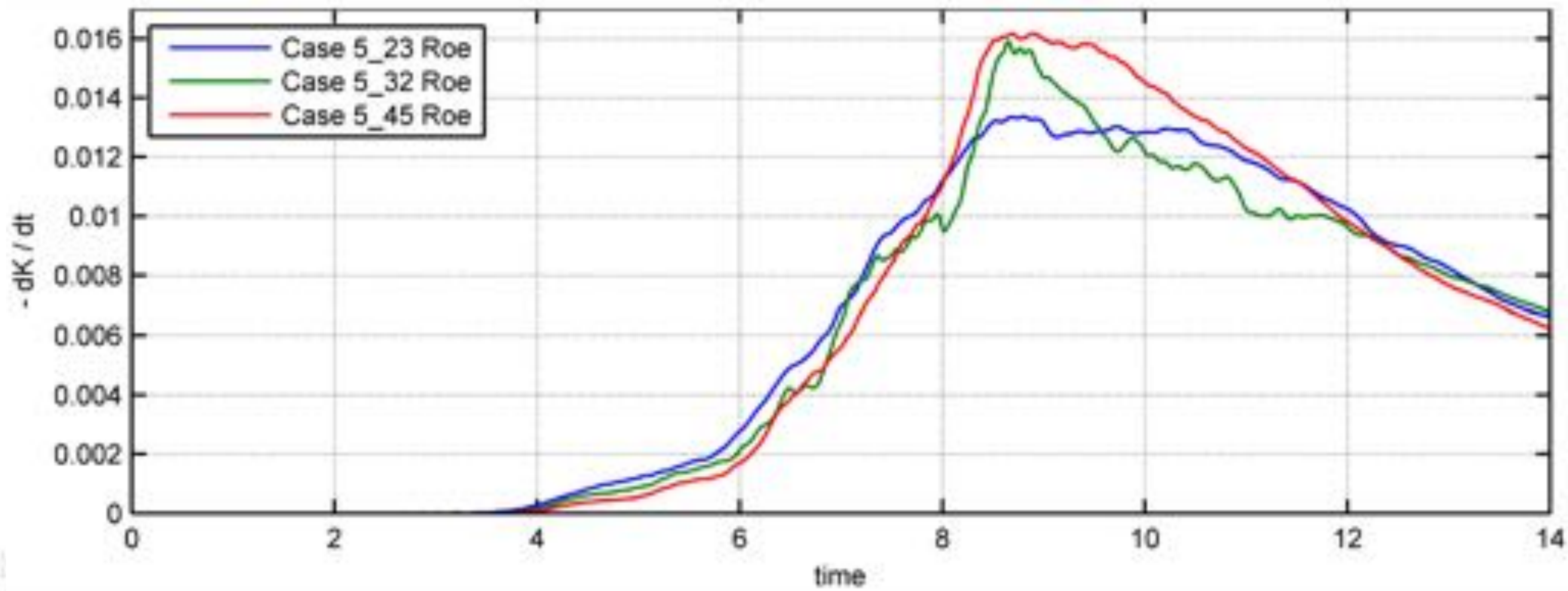


Summary of test cases — crossed out numbers indicate cases that crashed

	Roe (~ HLLC, Exact)					Lax-Friedrichs (~ HLL)				
$m = p + 1$	4	5	6	7	8	4	5	6	7	8
	28	23	19	16	14	28	23	~~19~~	~~16~~	~~14~~
n_{el}	39	32	28	23	~~19~~	39	32	~~28~~	~~23~~	~~19~~
	56	45	39	~~32~~	~~28~~	56	45	~~39~~	~~32~~	~~28~~



Numerical experiments with Nektar++ (inviscid TGV)



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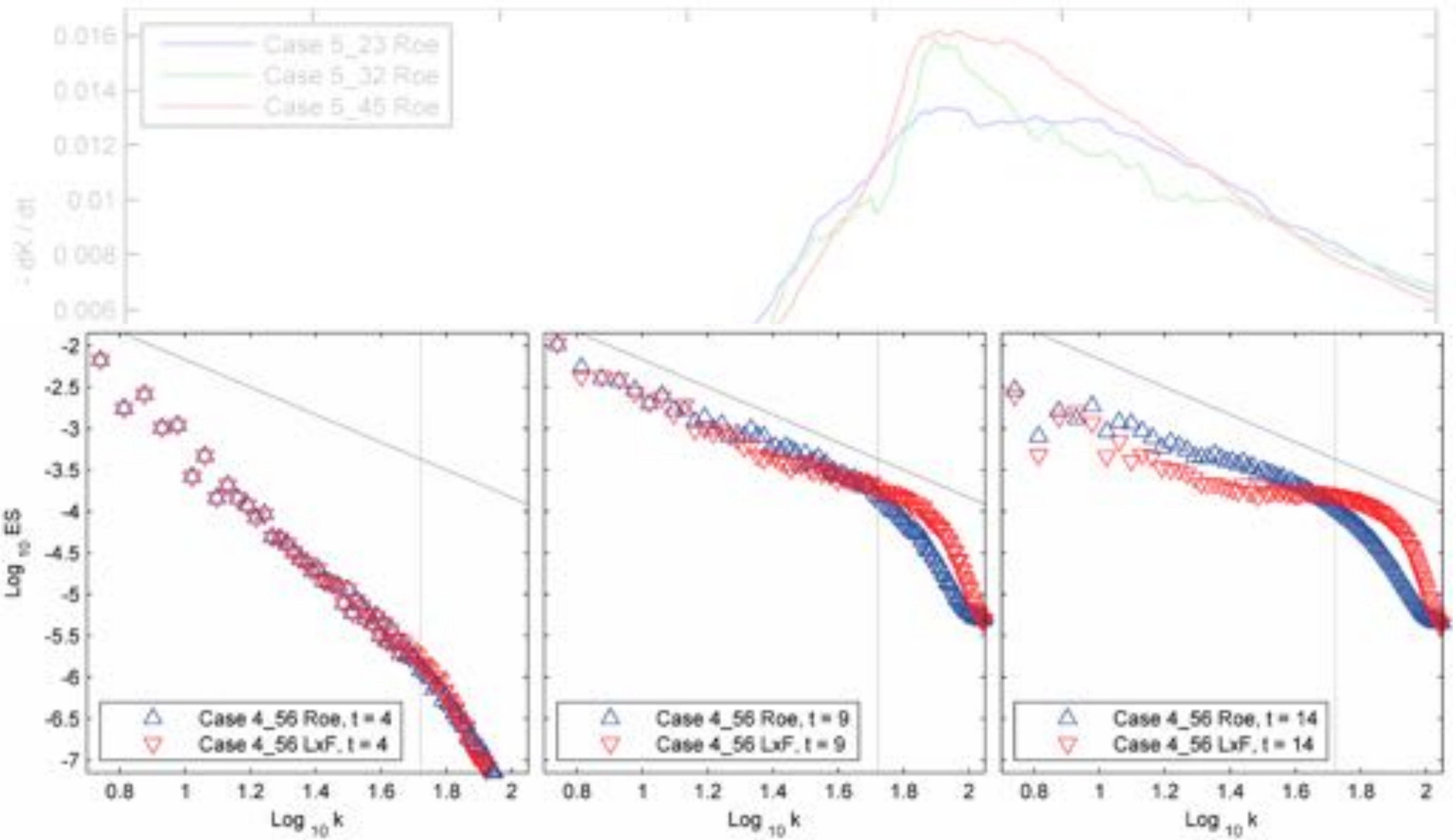
Numerical experiments with Nektar++ （inviscid TGV）



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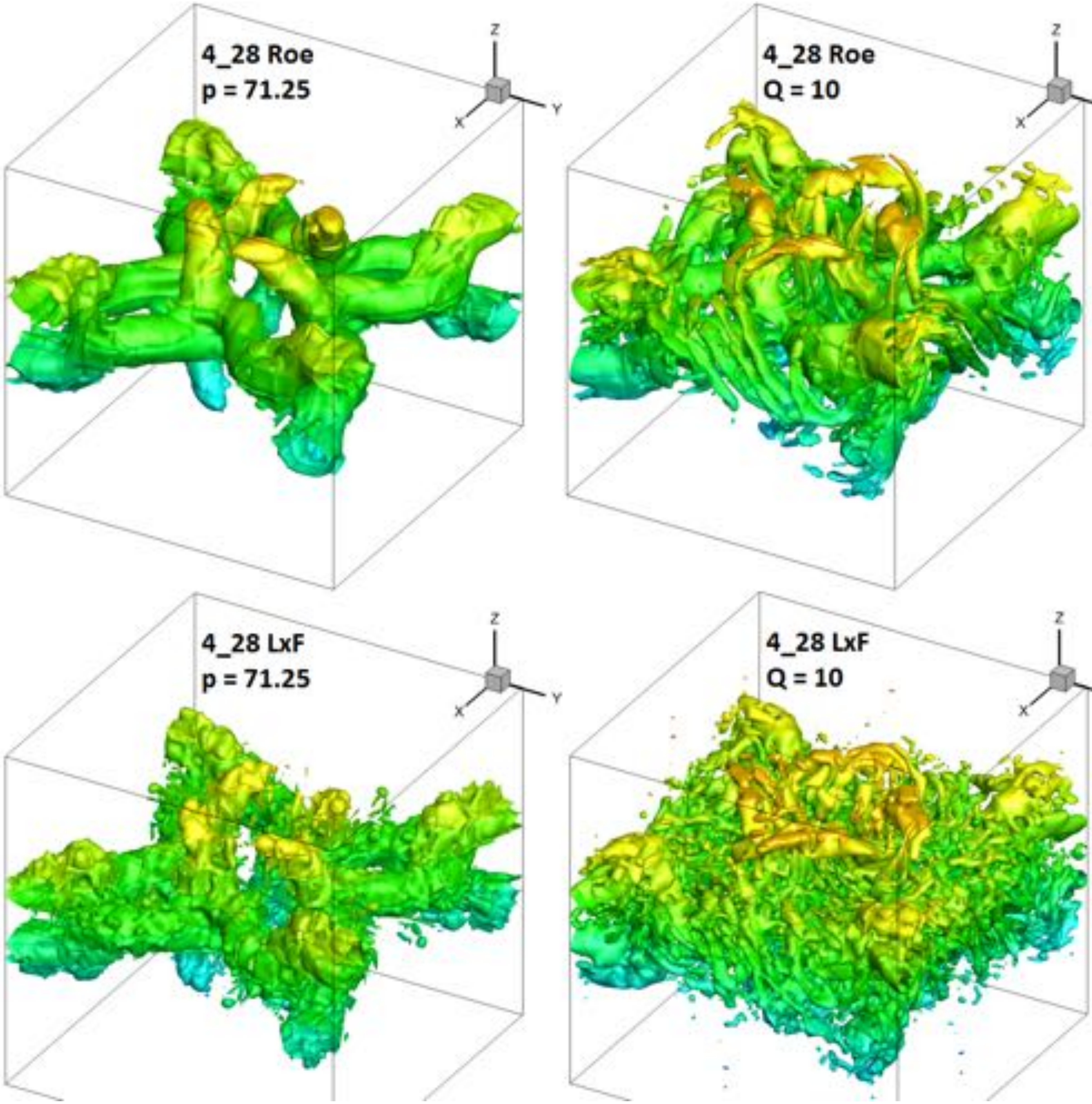

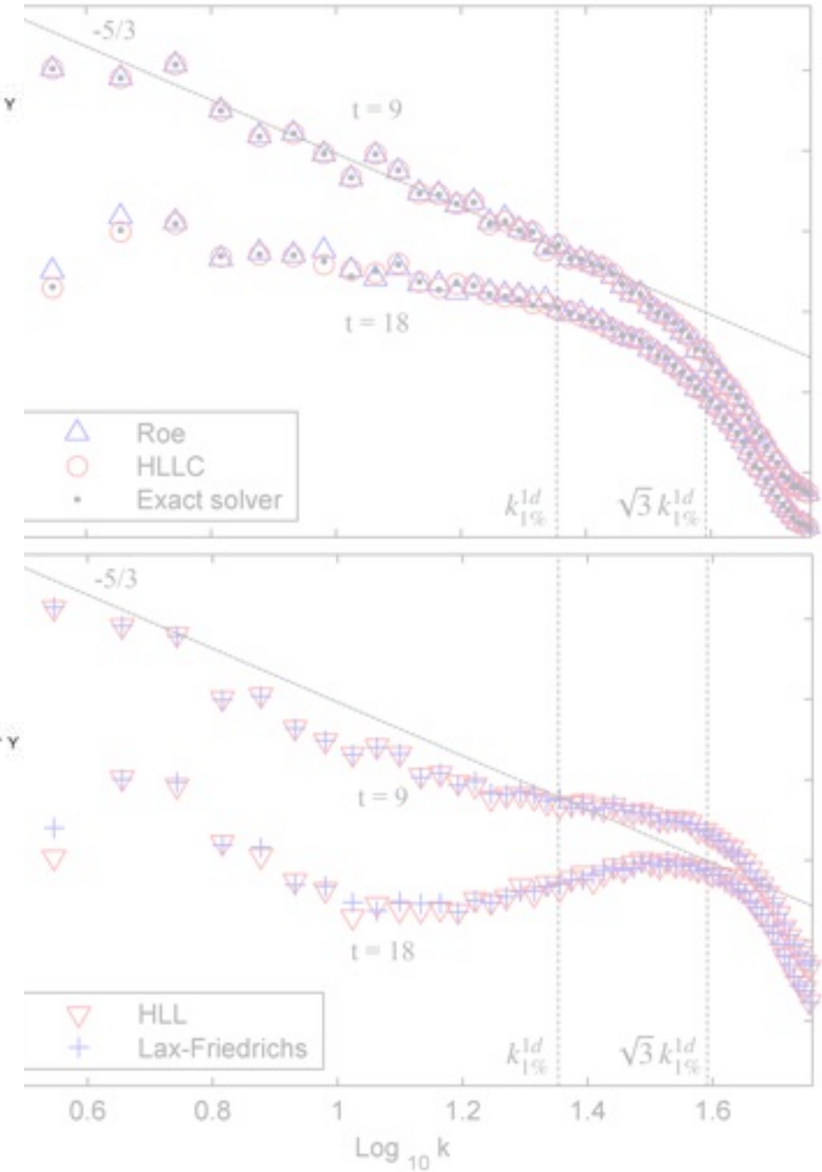

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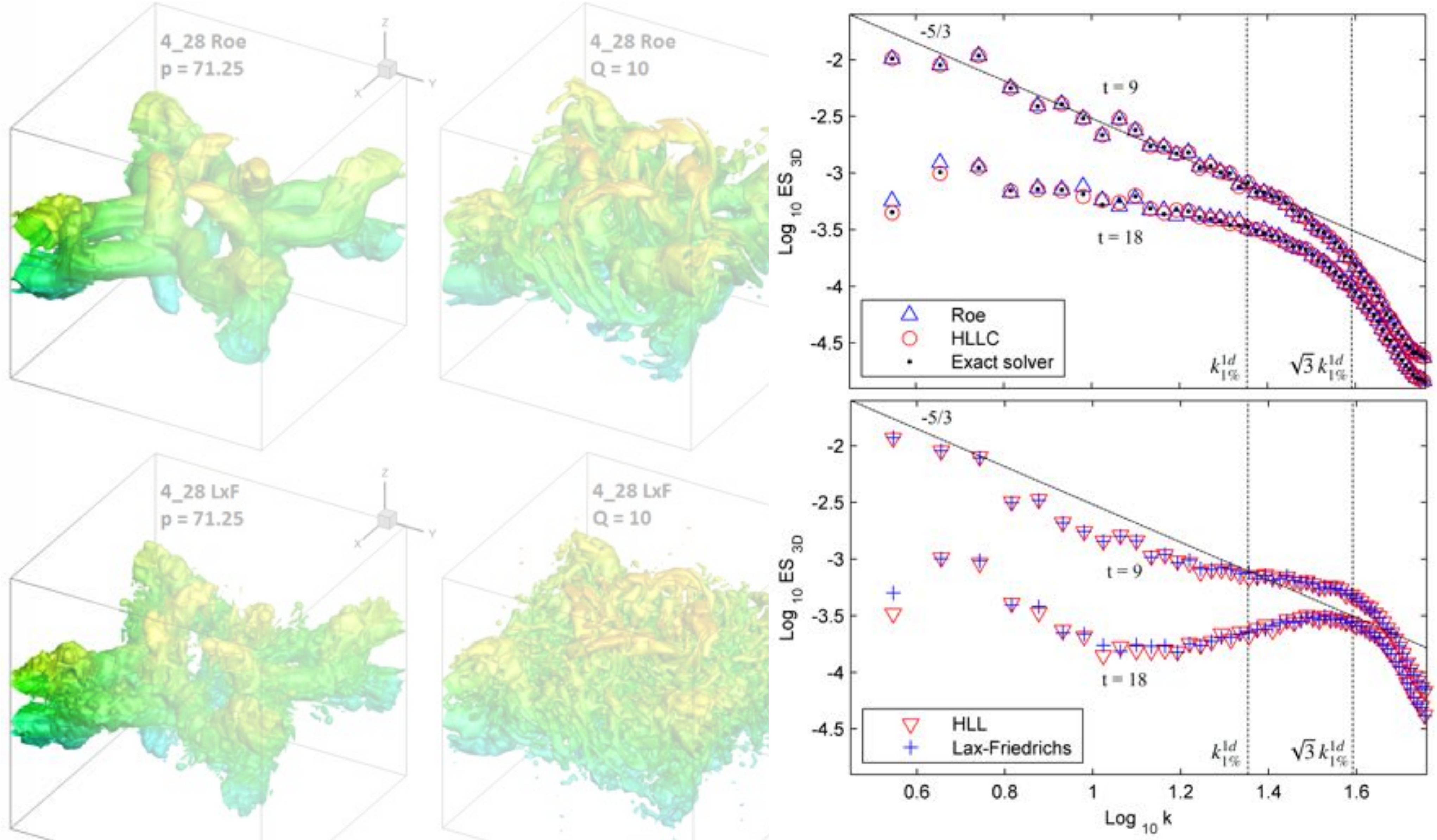
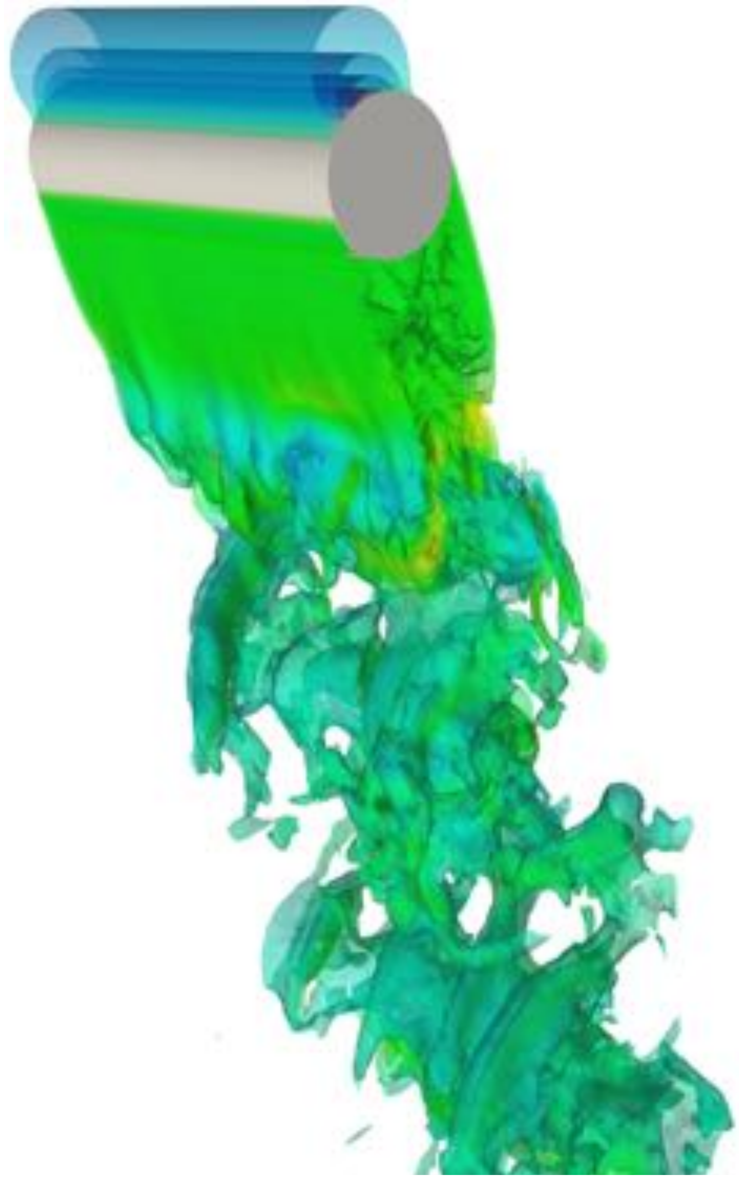


Conclusions & outlook

- Why does SEM-based iLES/uDNS work ?
- How to follow this approach ?
- 1% rule as estimate of "implicit" filter width
- Favor high-order with coarser meshes
- Stabilizing techniques at high Reynolds
- Avoid simplistic Riemann fluxes with DG
- Employ well-behaved SVV operators with CG



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Questions

Questions

MOURA, R.C.; MENGALDO, G.; SHERWIN, S.J.; PEIRÓ, J.: *On the eddy-resolving capability of high-order discontinuous Galerkin approaches to implicit LES / under-resolved DNS of Euler turbulence*. JCP (under review), 2016.

MOURA, R.C.; SHERWIN, S.J.; PEIRÓ, J.: *Eigensolution analysis of spectral/hp continuous Galerkin approximations to advection-diffusion problems: insights into spectral vanishing viscosity*. JCP, v. 307, p. 401-422, 2016.

MOURA, R.C.; SHERWIN, S.J.; PEIRÓ, J.: *Linear dispersion-diffusion analysis and its application to under-resolved turbulence simulations using discontinuous Galerkin spectral/hp methods*. JCP, v. 298, p. 695-710, 2015.

MOURA, R.C.; SHERWIN, S.J.; PEIRÓ, J.: *Modified Equation Analysis for the Discontinuous Galerkin Formulation*. In: ICOSAHOM, 2014 (Lecture Notes in Computational Science and Engineering, 2015).