Suitability of Artificial Viscosity Discontinuous Galerkin Method for Compressible Turbulence

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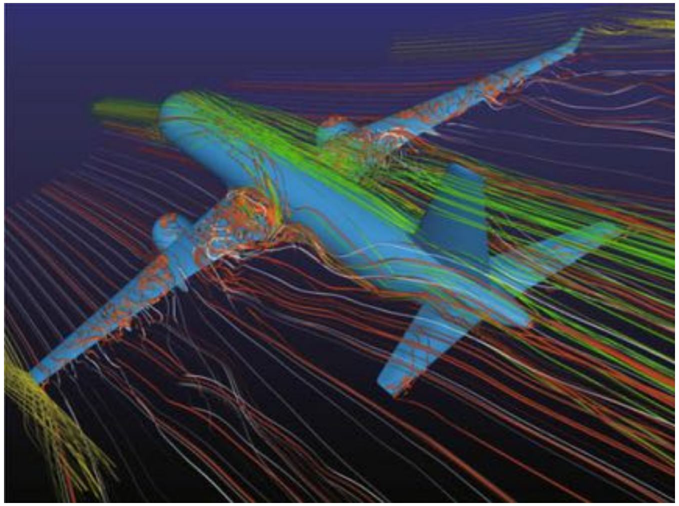
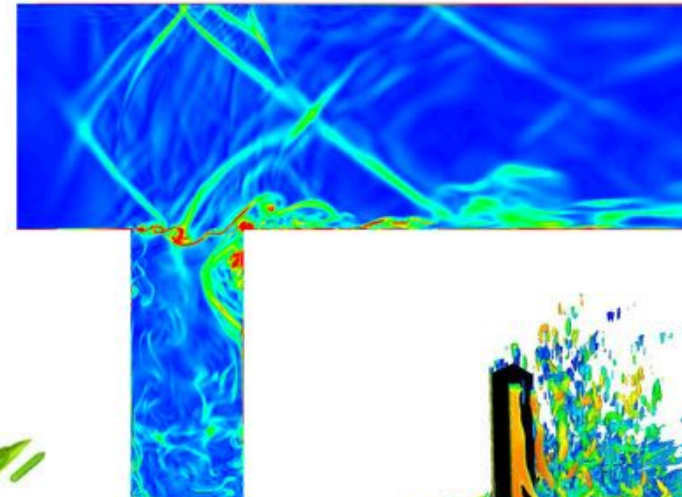
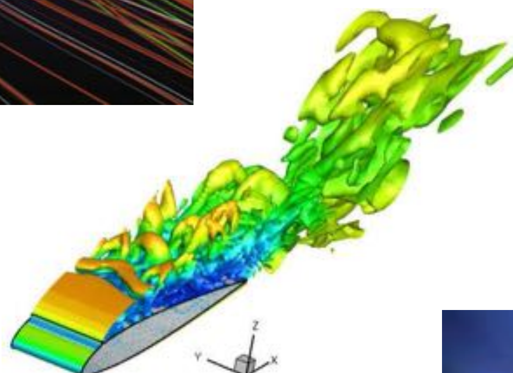
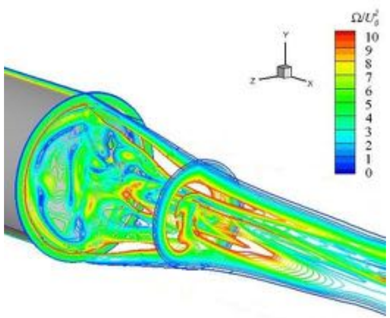
Motivation

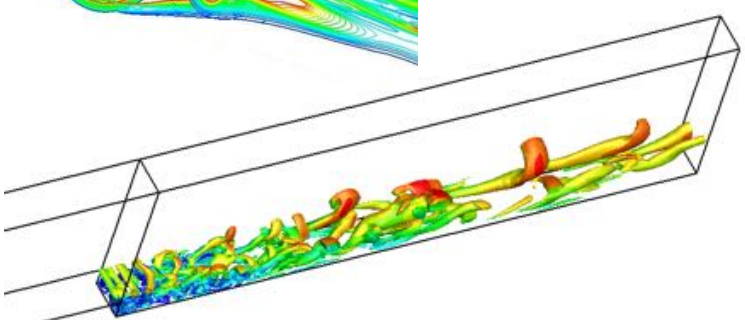
Complex Configuration & Complex Flow Sturctures











Motivation

- Implicit LES frequently employed when DG is considered for LES-type problems
  - ✓ Quantification of numerical dissipation to be addressed
  - ✓ Most work in the literature focus on flows with no shocks
- Dealiasing for under-resolved solutions
- Shock capturing methods
  - ✓ Sub-cell resolution ability
  - ✓ Effects on broadband accuracy
  - ✓ Effects on aliasing effects
- Performance relative to high order finite difference methods remains to be further addressed

Method

- Compressible Flow Solver within the Nektar++ framework
- Govening equations
  - ✓ EulerADCFE
  - ✓ NavierStokesADCFE (Added through mimicking EulerADCFE and NavierStokesCFE)
- Element type: Quad, Hex
- Discretization: Weak DG
- Numerical flux: HLLC
- Time integration: ClassicalRungeKutta4

Method

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- Shock capturing: NonSmooth
- Modal Sensor- Persson and Peraire's AIAA Paper, 2006

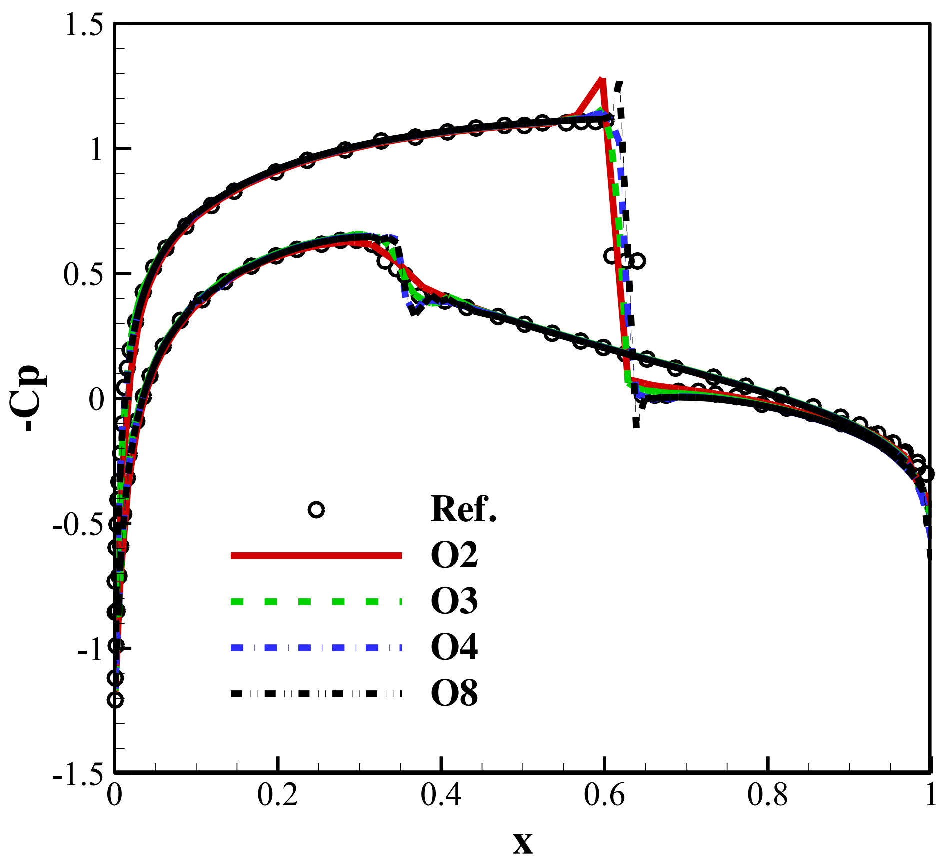
$$\mu_{\mathrm{av}} = \varepsilon_0 \begin{cases} 0 & S_e < s_0 - \kappa \\ \frac{1}{2}\left(1 + \sin\left(\frac{\pi(S_e - s_0)}{2\kappa}\right)\right), & s_0 - \kappa \le S_e \le s_0 + \kappa \\ 1 & S_e > s_0 + \kappa \end{cases}$$

$$S_e = \log_{10}\left(\frac{\langle q_h - \bar{q}_h, q_h - \bar{q}_h \rangle_e}{\langle q_h, q_h \rangle_e}\right), \qquad \bar{q}_h = \sum_{n=1}^{N(O-1)} \hat{u}_{n,e}(t)\varphi_n(\mathbf{x})$$

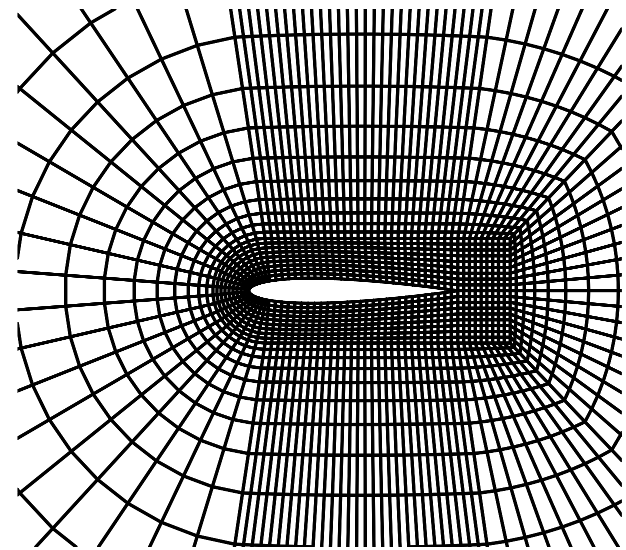
- ε_0 governs the vaule of the artificial viscosity, while s_0 determines the action range of the artificial viscosity
- ε_0 and s_0 remain constant for each test case unless specified otherwise

Inviscid Transonic NACA 0012 Flow

- Ma=0.8, α=1.25
- Shock-dominated inviscid flows



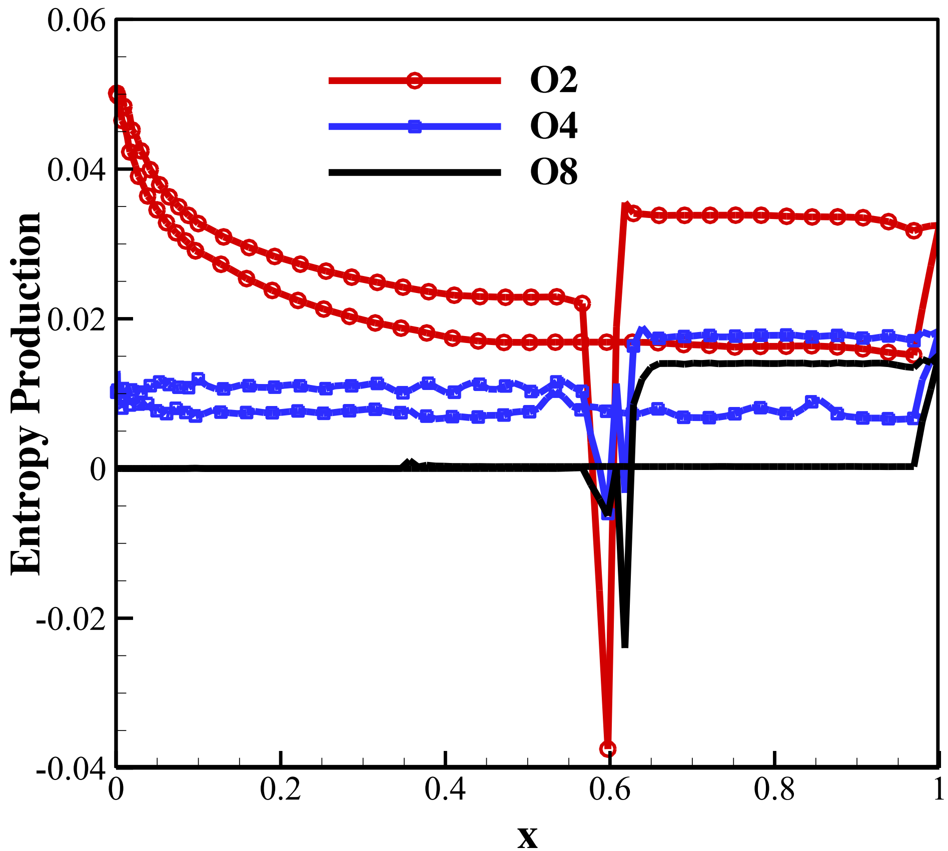
Enlarged mesh (Taken from tutorials)



Validation with different orders

*Ref taken from Luo et al, JCP,2007

Inviscid Transonic NACA 0012 Flow

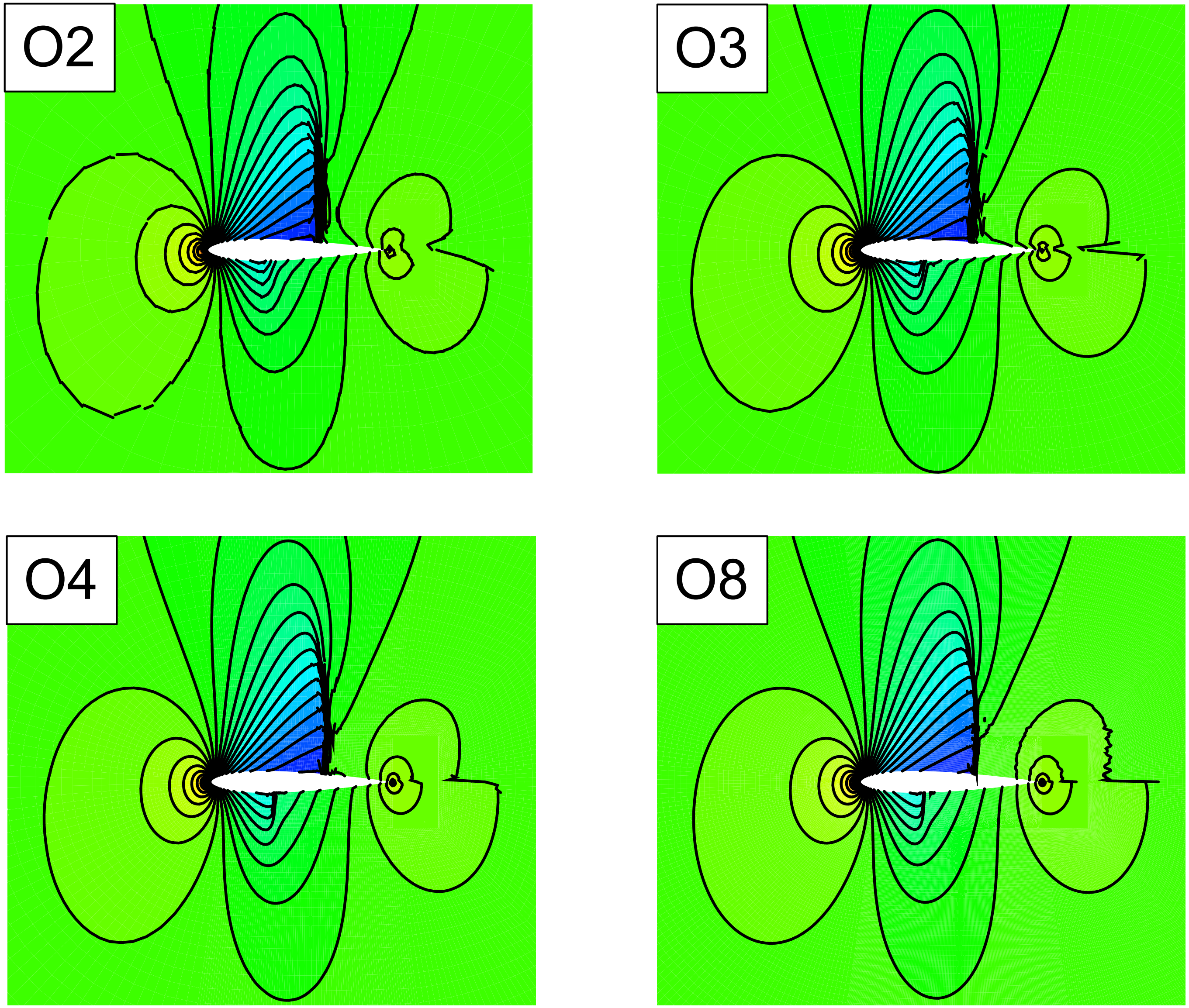


- Higher order schemes generate lower entropy production as expected

Inviscid Transonic NACA 0012 Flow

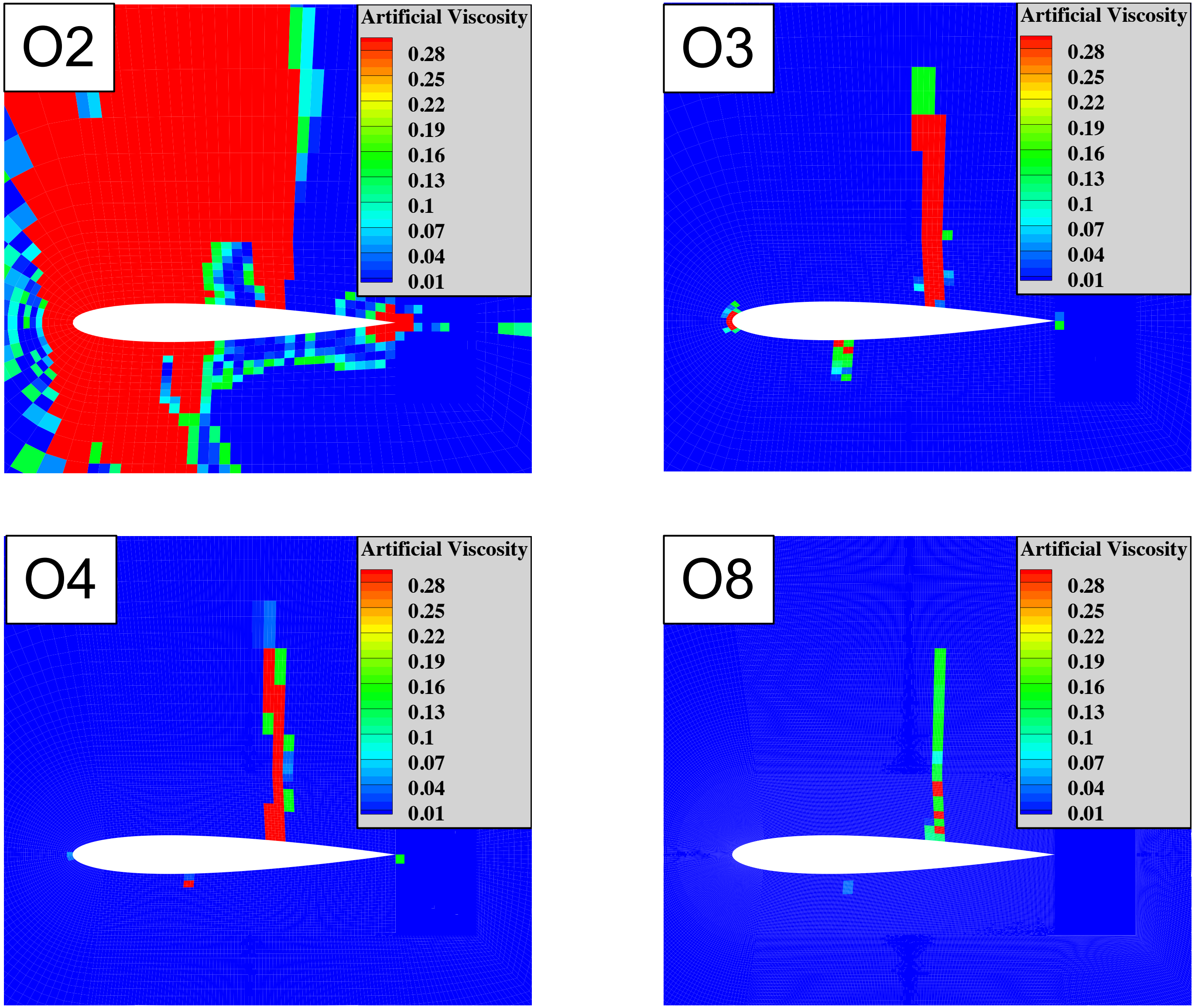
Density

Contours



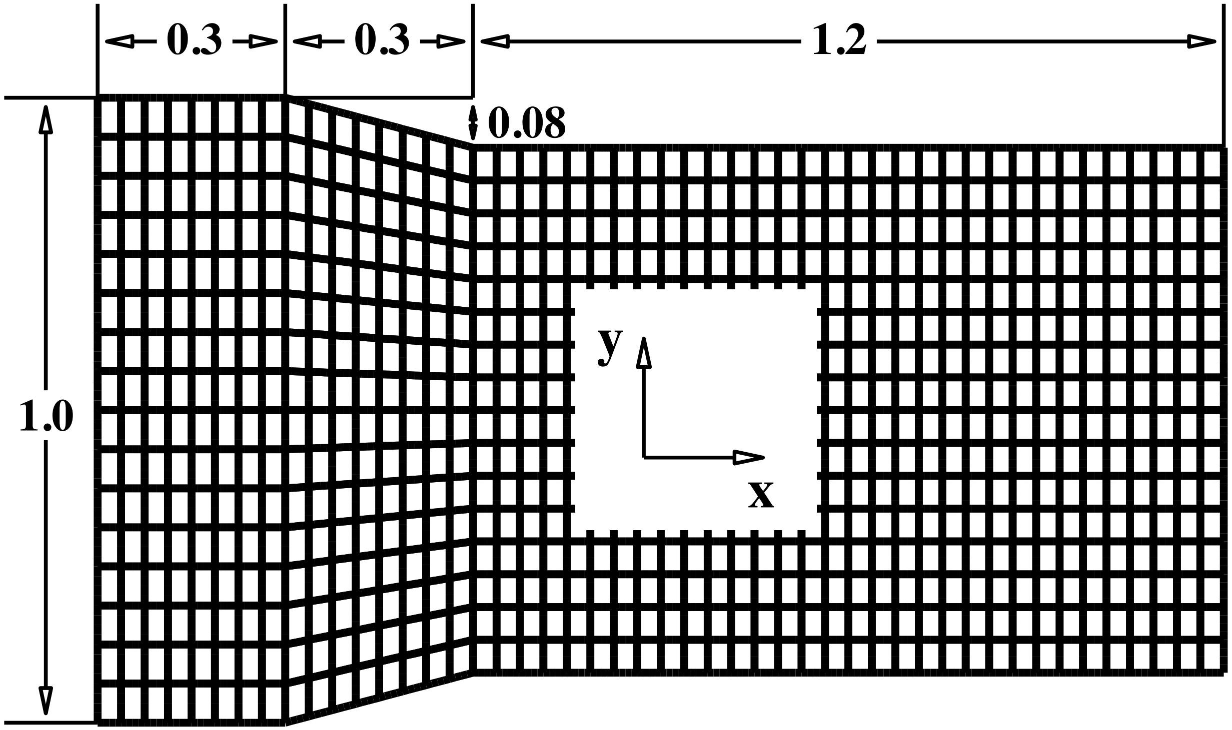
Inviscid Transonic NACA 0012 Flow

Artificial Viscosity

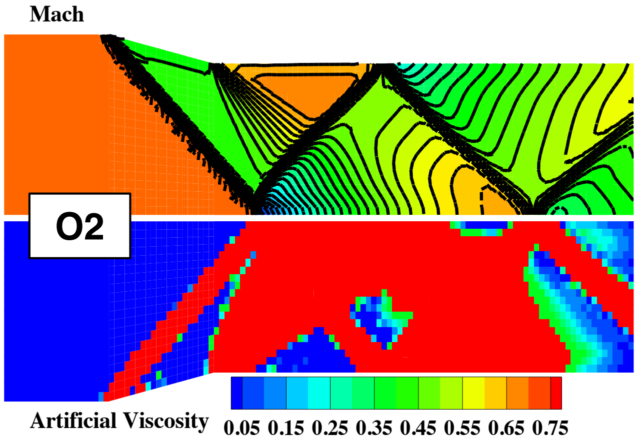


Inviscid Supersonic Tube Flow

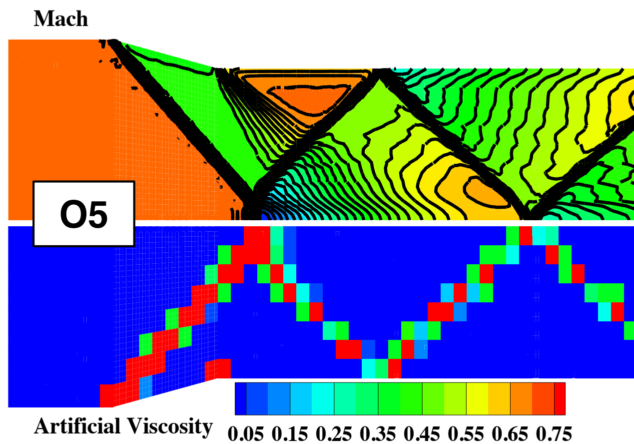
- Inflow Mach number: 1.9
- Shock-dominated inviscid flows
- Constant DOFs

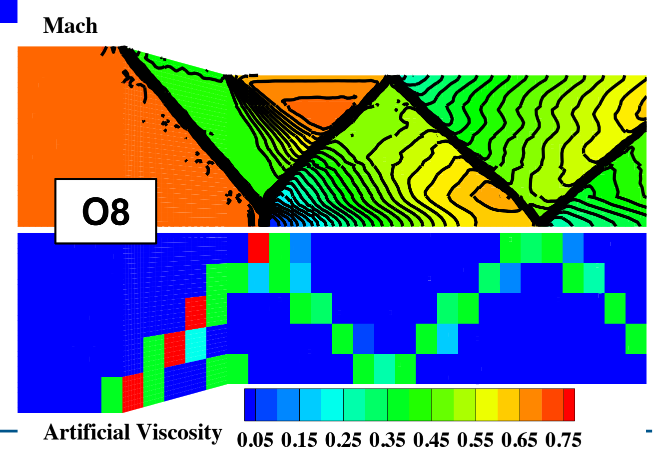


Inviscid Supersonic Tube Flow



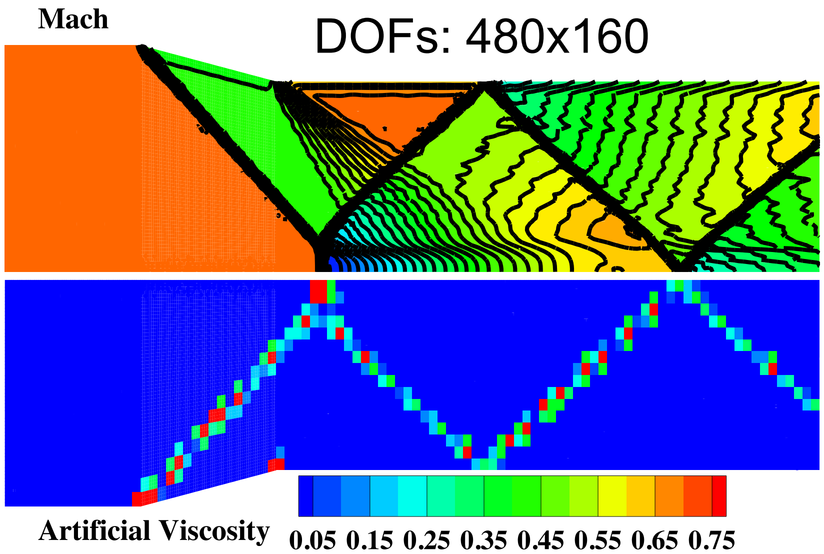
- Constant DOFs: 240x80

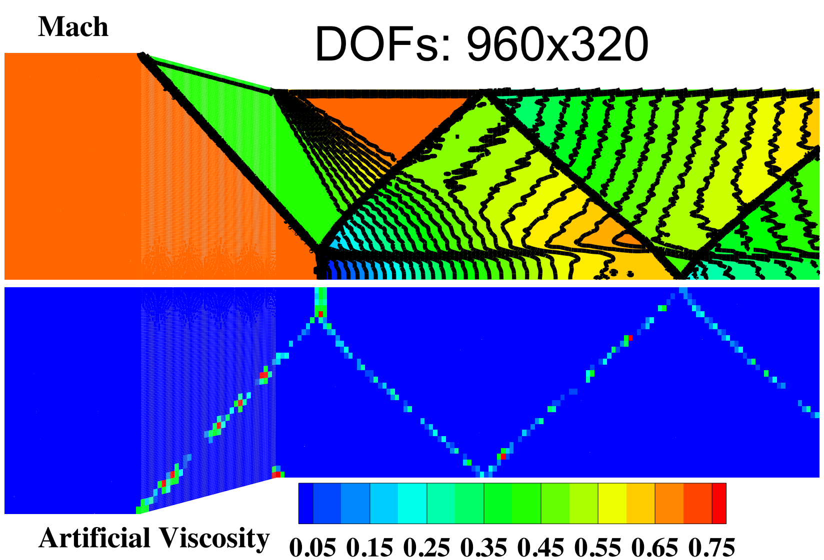




- Sharper shocks resolved with higher accuracy orders

Inviscid Supersonic Tube Flow

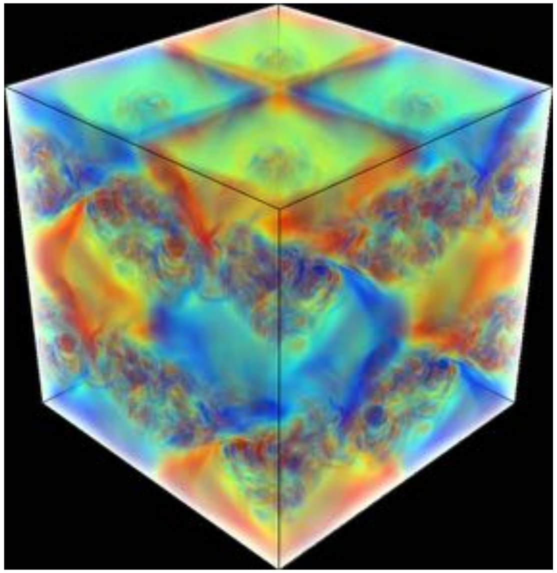




- Fifth order AVDG
- The empirical parameters for the shock capturing are kept the same as those on the baseline resolution

Taylor Green Vortex Flow

- Undergoes transition to fully turbulence
- Mach number: 0.1
- Reynolds number: 1600
- Artificial viscosity applied
- Initial conditions:



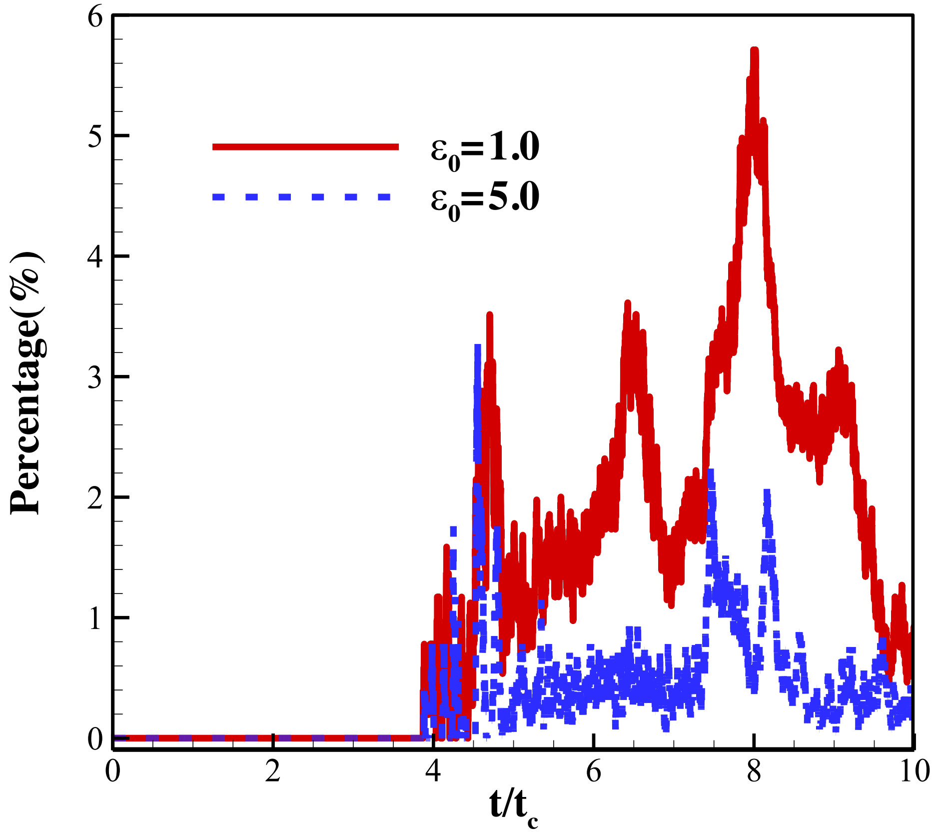
$$u_1 = V_0 \sin(\frac{x}{L}) \cos(\frac{y}{L}) \cos(\frac{z}{L})$$

$$u_2 = -V_0 \cos(\frac{x}{L}) \sin(\frac{y}{L}) \cos(\frac{z}{L})$$

$$u_3 = 0.0$$

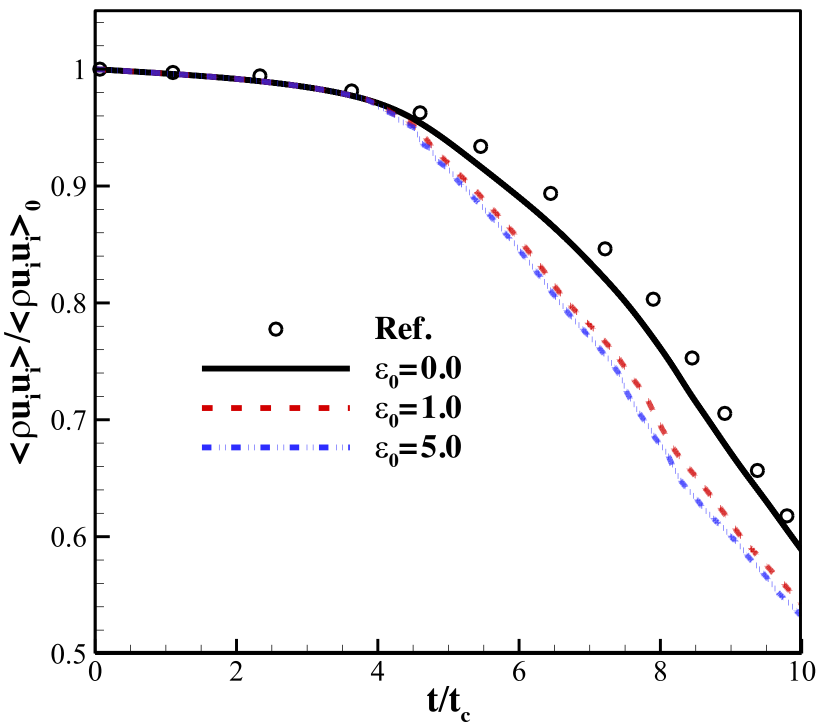
$$p = P_0 + \frac{\rho_0 V_0^2}{16}\left[\cos(\frac{2x}{L}) + \cos(\frac{2y}{L})\right]\left[\cos(\frac{2z}{L}) + 2\right]$$

- Fourth order AVDG,DOFs:64^3
- Less elements are polluted by the artificial viscosity with a larger ε_0 as expected

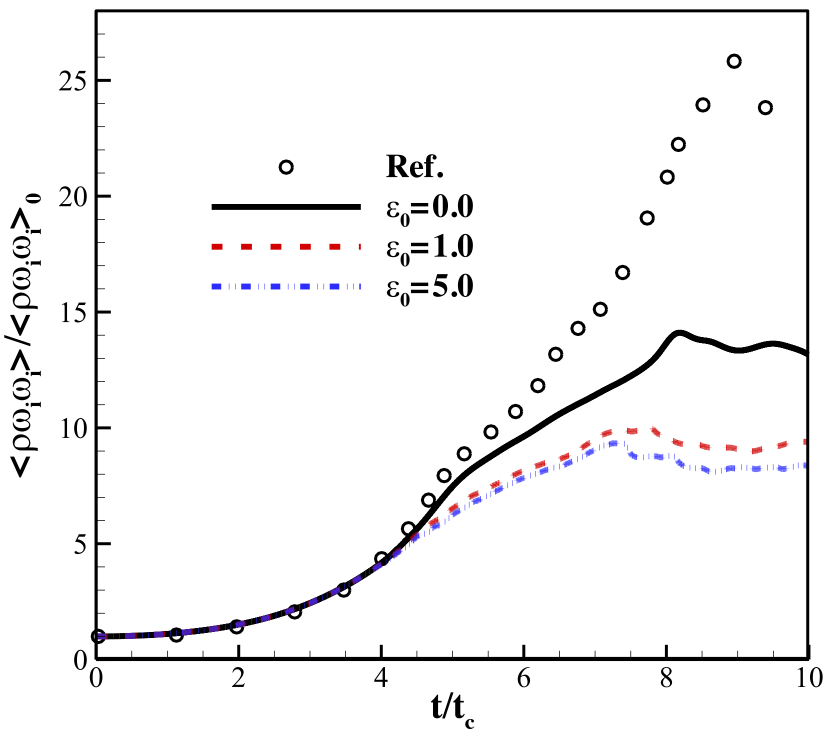


Percentage of elements where artificial viscosity is non-zero

Kinetic energy

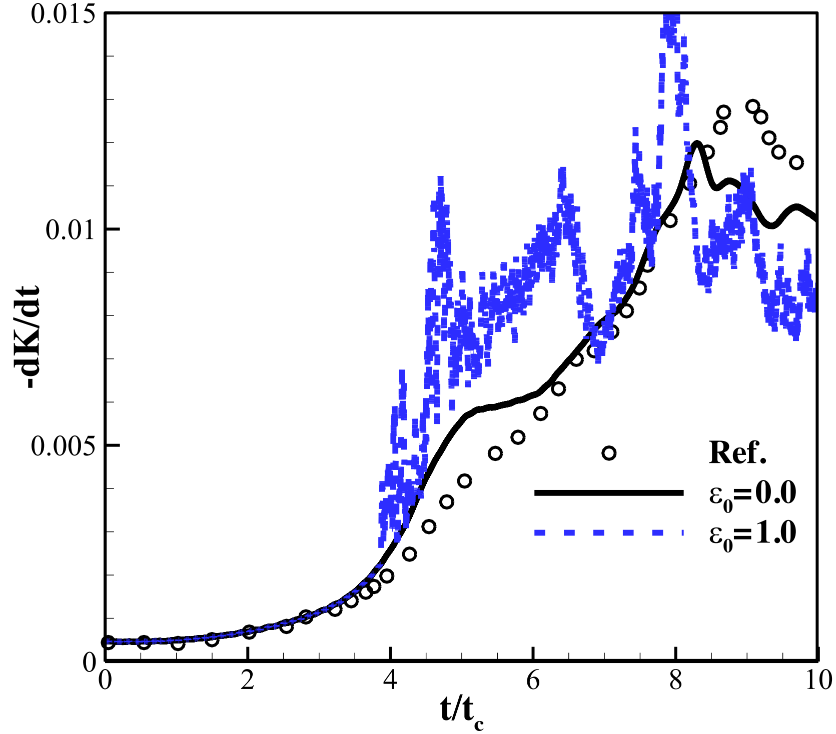


Enstrophy

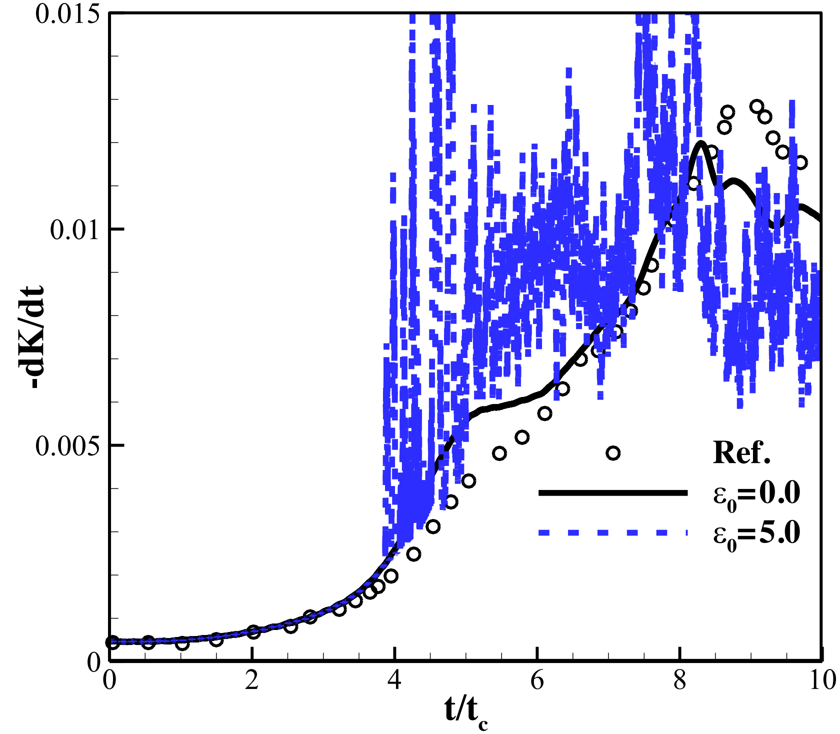


- Detrimental effect of artificial dissipation noticeable
- Little difference between different non-zero ε_0

ε_0=1.0



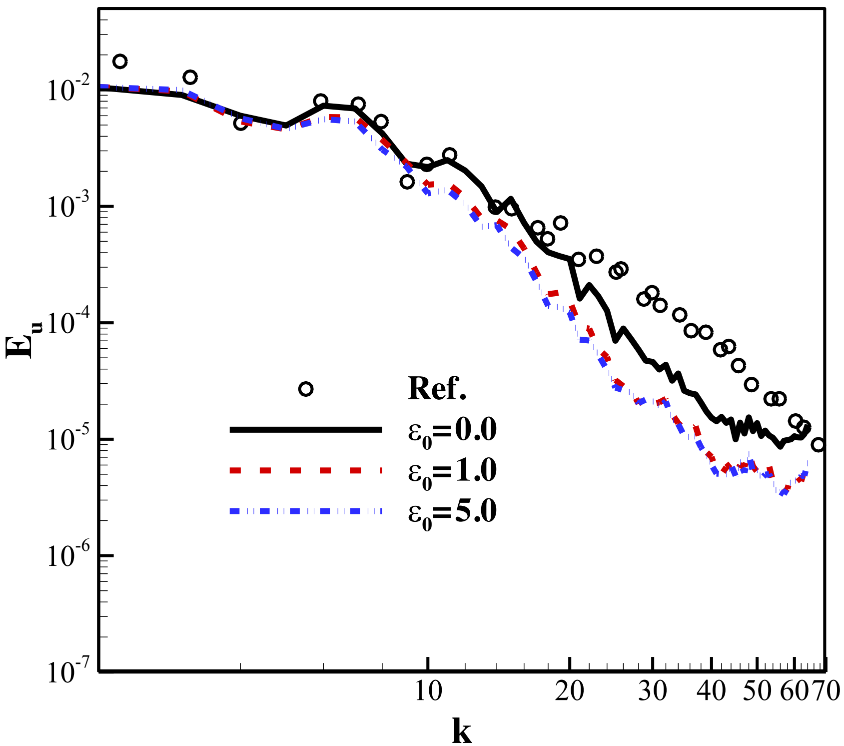
ε_0=5.0

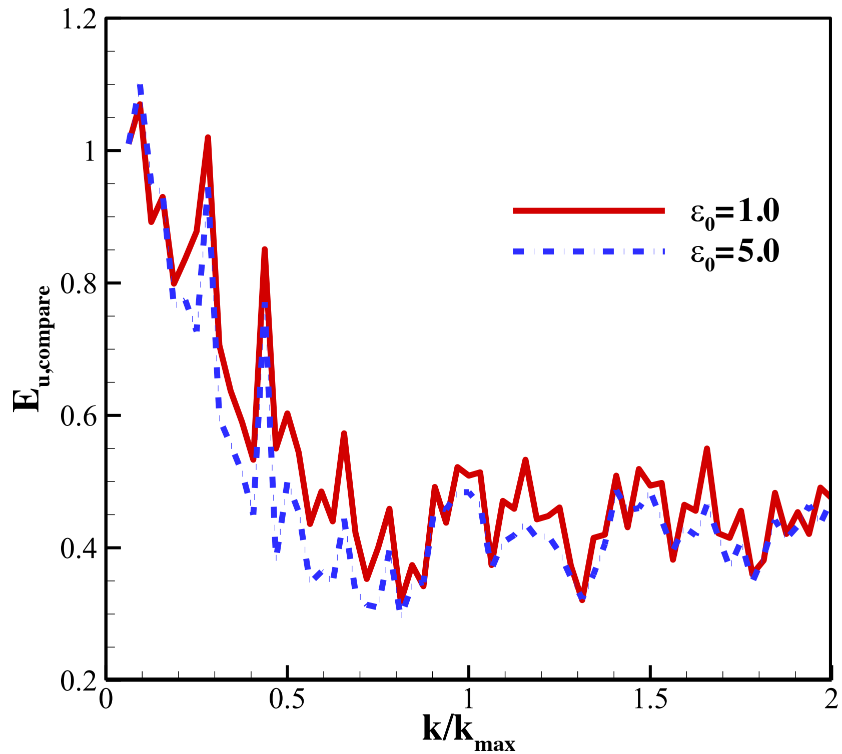


- Oscillatory behavior due to numerical dissipation

- Energy spectrum at $t = 9t_c$

$$E_{u,\text{compare}}(k) = \frac{E_{u,\varepsilon_0}(k)}{E_{u,\varepsilon_0=0.0}(k)}$$



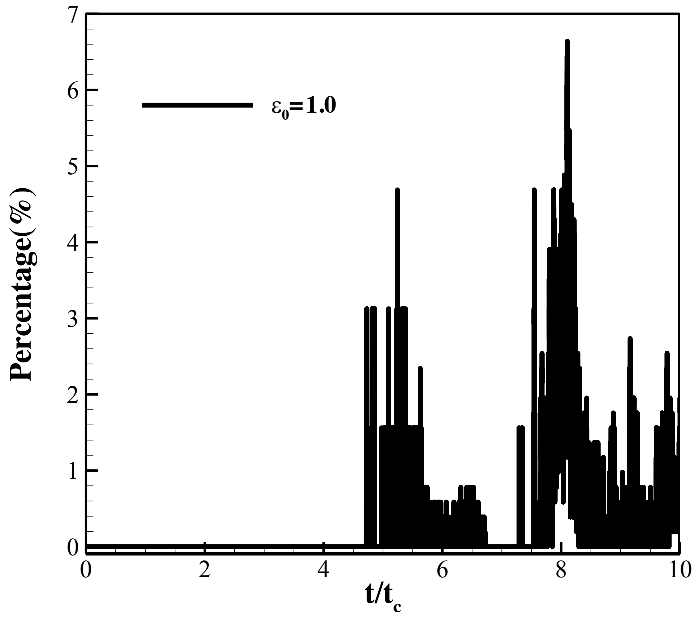


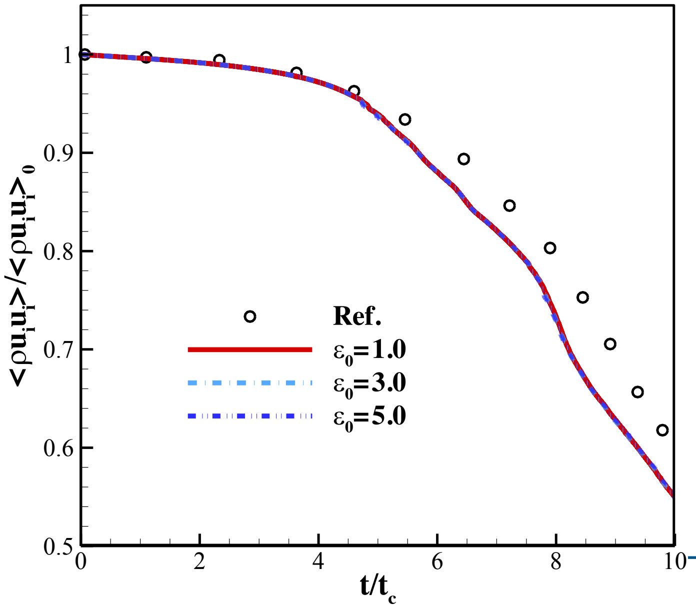
$k_{\max} = 32$

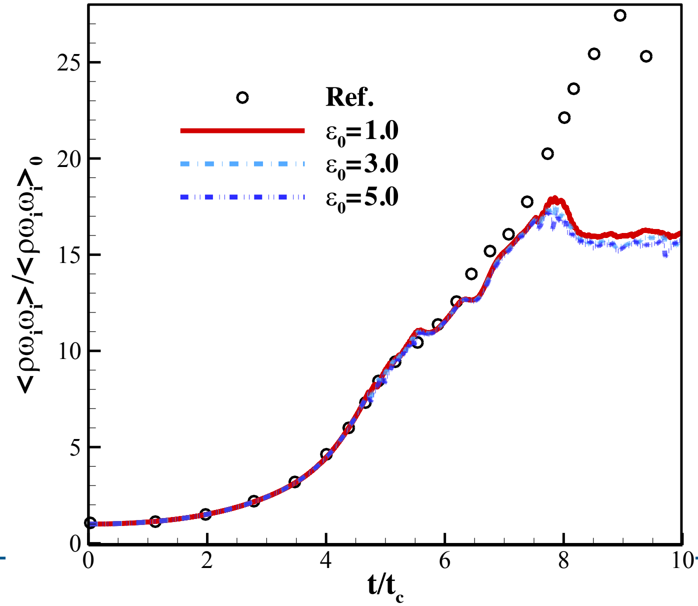
- Effect of artificial dissipation biased towards higher wave numbers

Effect of artificial viscosity for underresovled instability

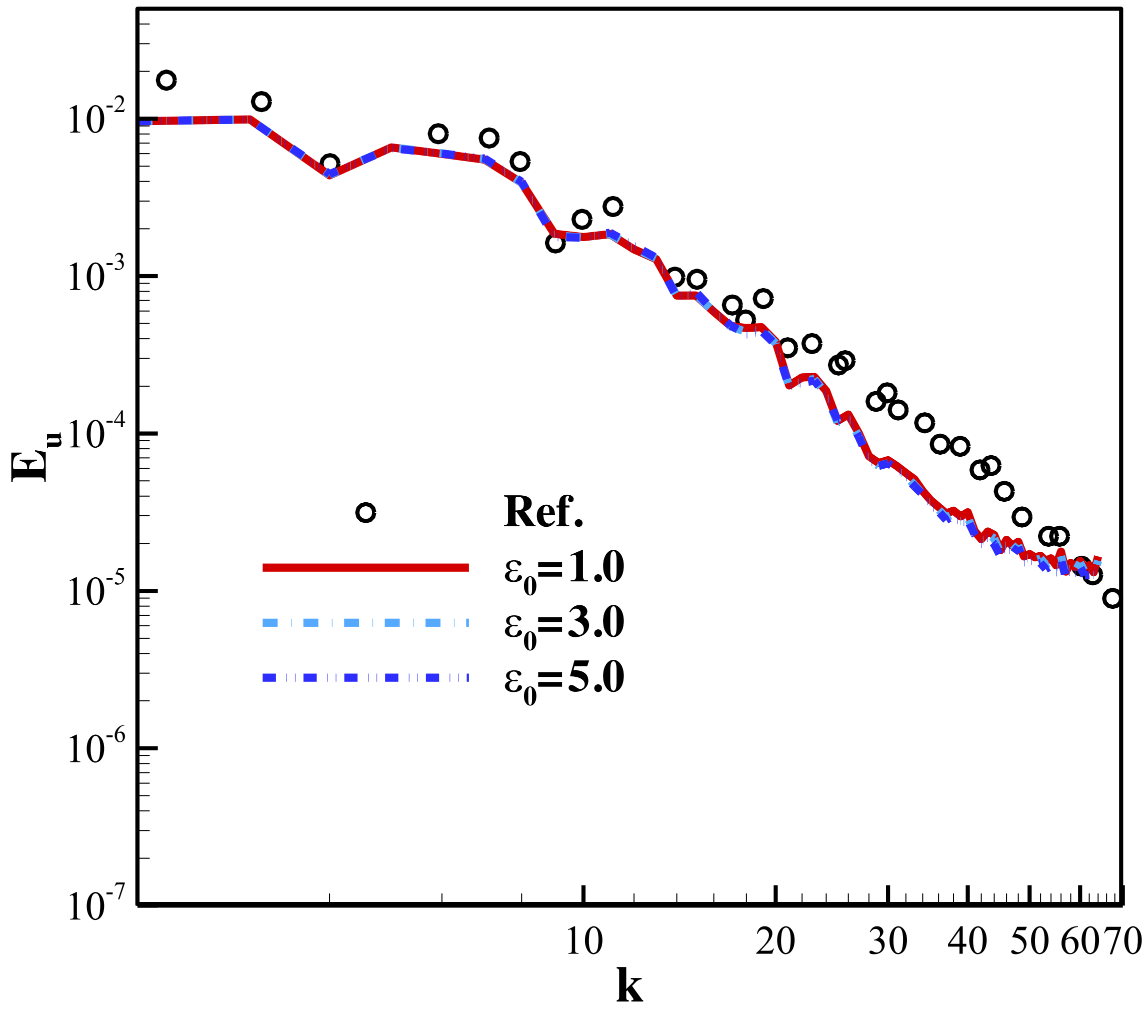
- The eighth order DG method would blow up due to aliasing effects
- With the artificial viscosity model, the computations run properly over the entire period of simulation





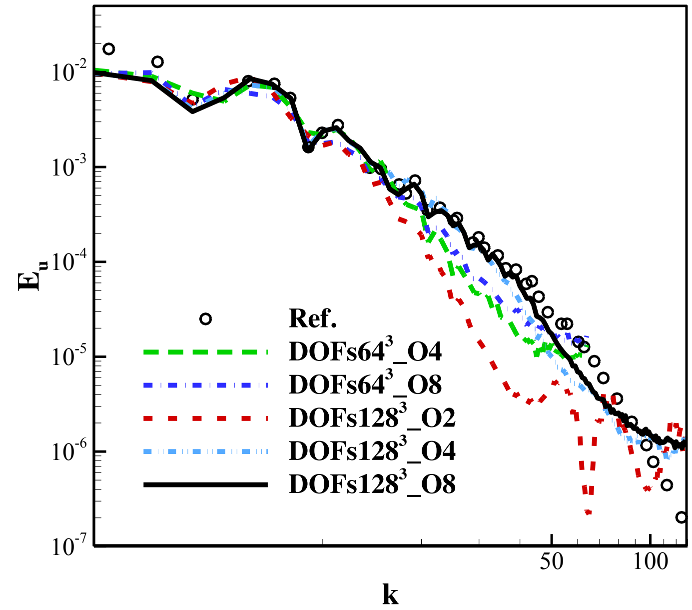


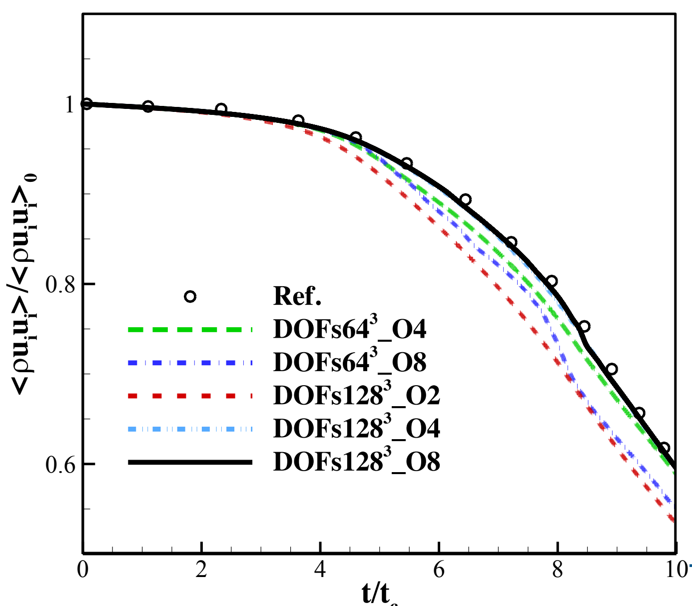
Effect of artificial viscosity for underresovled instability

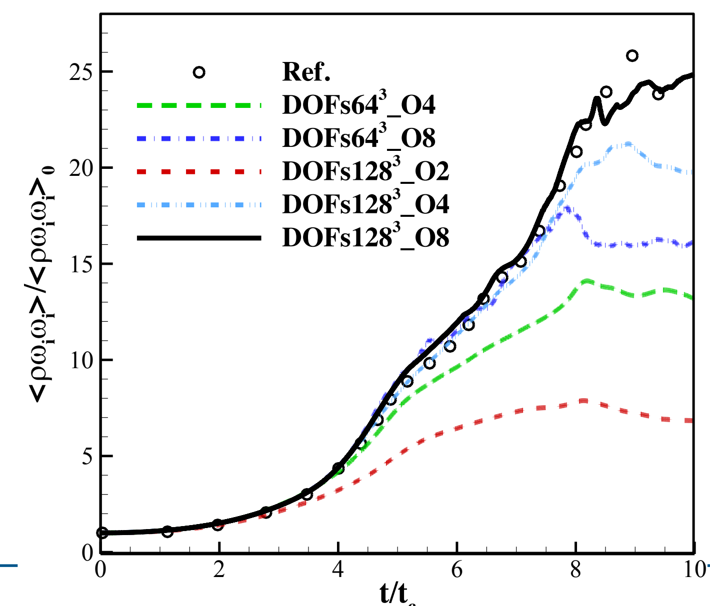


Effect of order and resolution on broadband accuracy

- AVDG for eighth order, DG for others
- Discrepancy of enstrophy more obvious
- Eighth order AVDG over fourth order DG

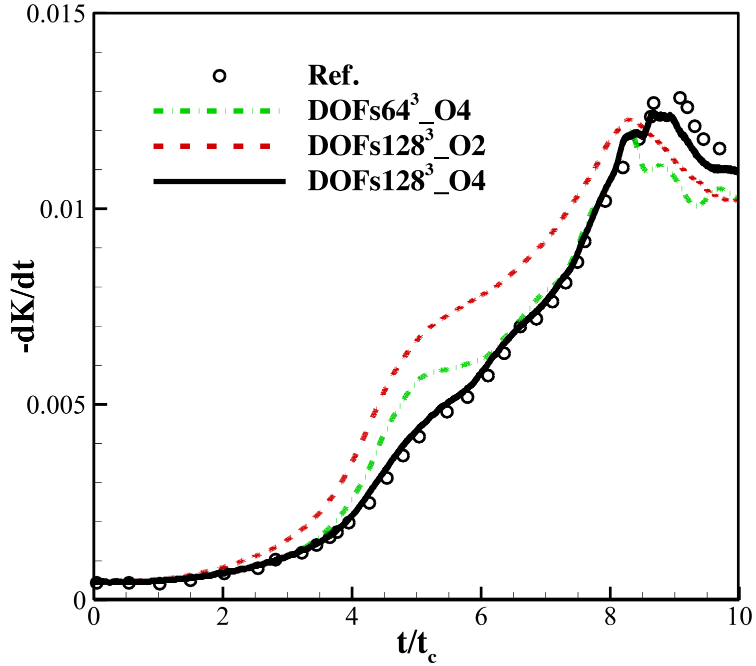






- For incompressible flow, the enstrophy is directly related to the kinetic energy dissipation rate through a constant, i.e.

$$-\frac{dK}{dt} \sim < \rho\omega_i\omega_i >$$



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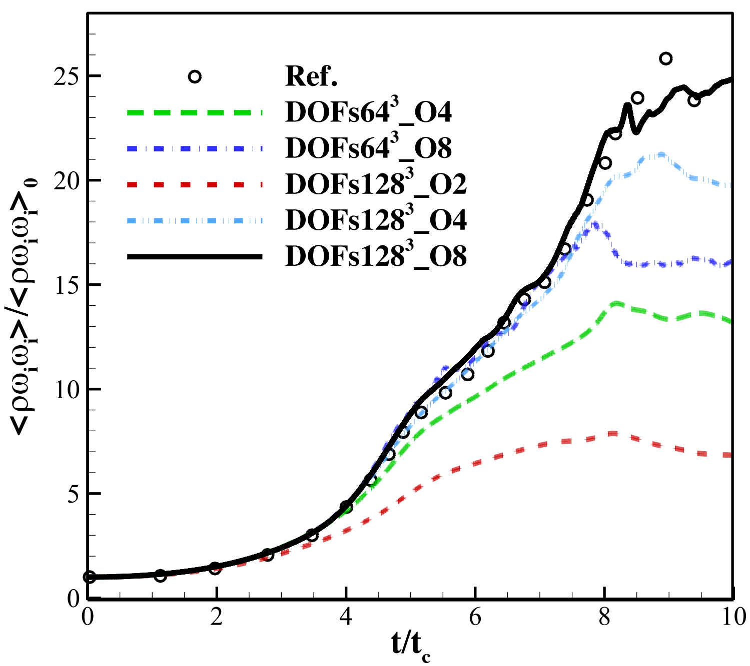


- Ingredients of the total dissipation rate are then decomposed as following*

$$\frac{dK}{dt} = \frac{d}{dt}\left\langle \frac{1}{2}\rho u_i u_i \right\rangle = -\left(\varepsilon_{\mathrm{d}} + \varepsilon_{\mathrm{s}}\right)$$

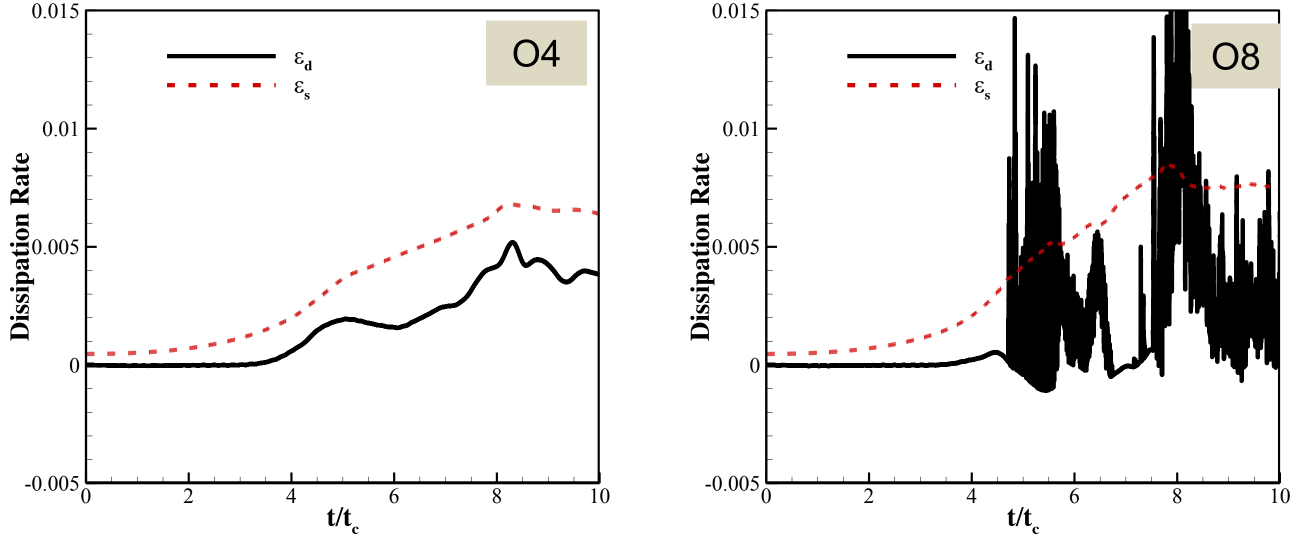
$$\varepsilon_{\mathrm{s}} = \left\langle 2\mu s_{i,j} s_{i,j} \right\rangle$$: solenoidal dissipation

$$\varepsilon_{\mathrm{d}}$$: numerical dissipation

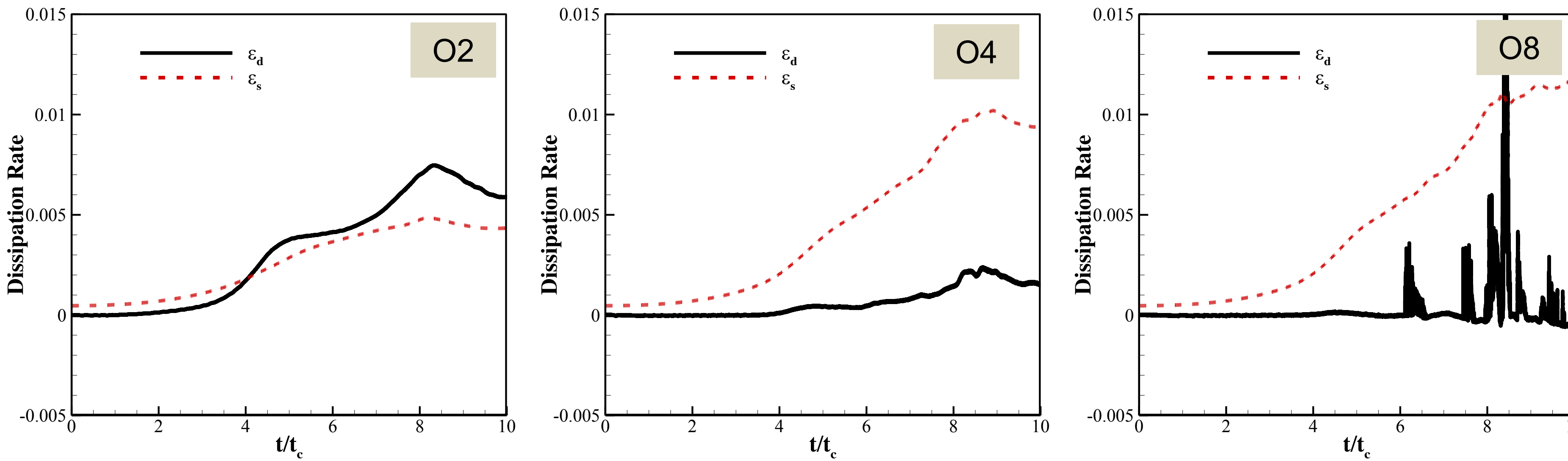
*Bull, J. R., and Jameson, A., "Simulation of the Compressible Taylor Green Vortex using High-Order Flux Reconstruction Schemes," AIAA Paper 2014-3210, Jun. 2014.

Effect of order and resolution on broadband accuracy

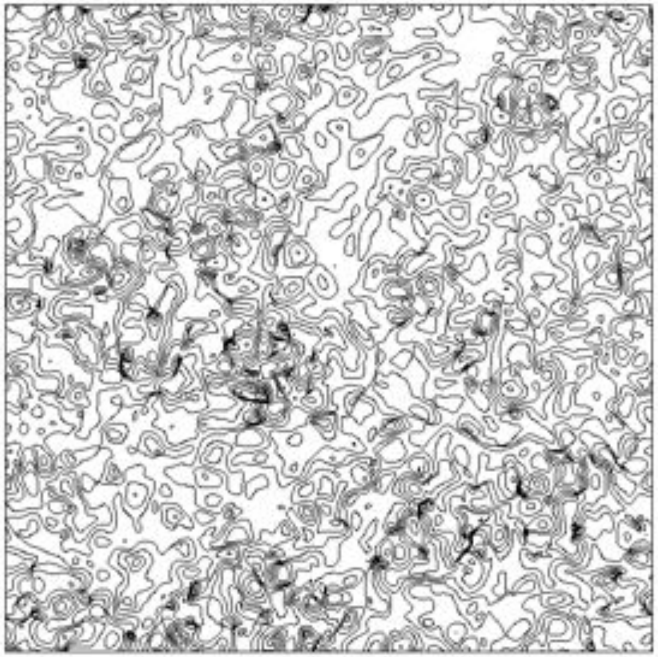
DOFs: 64^3



DOFs: 128^3



Compressible Isotropic Decaying Turbulence



- $Ma_{t,0} = 0.6 \quad \mathrm{Re}_{\lambda,0} = 100$
- Initial energy spectrum*

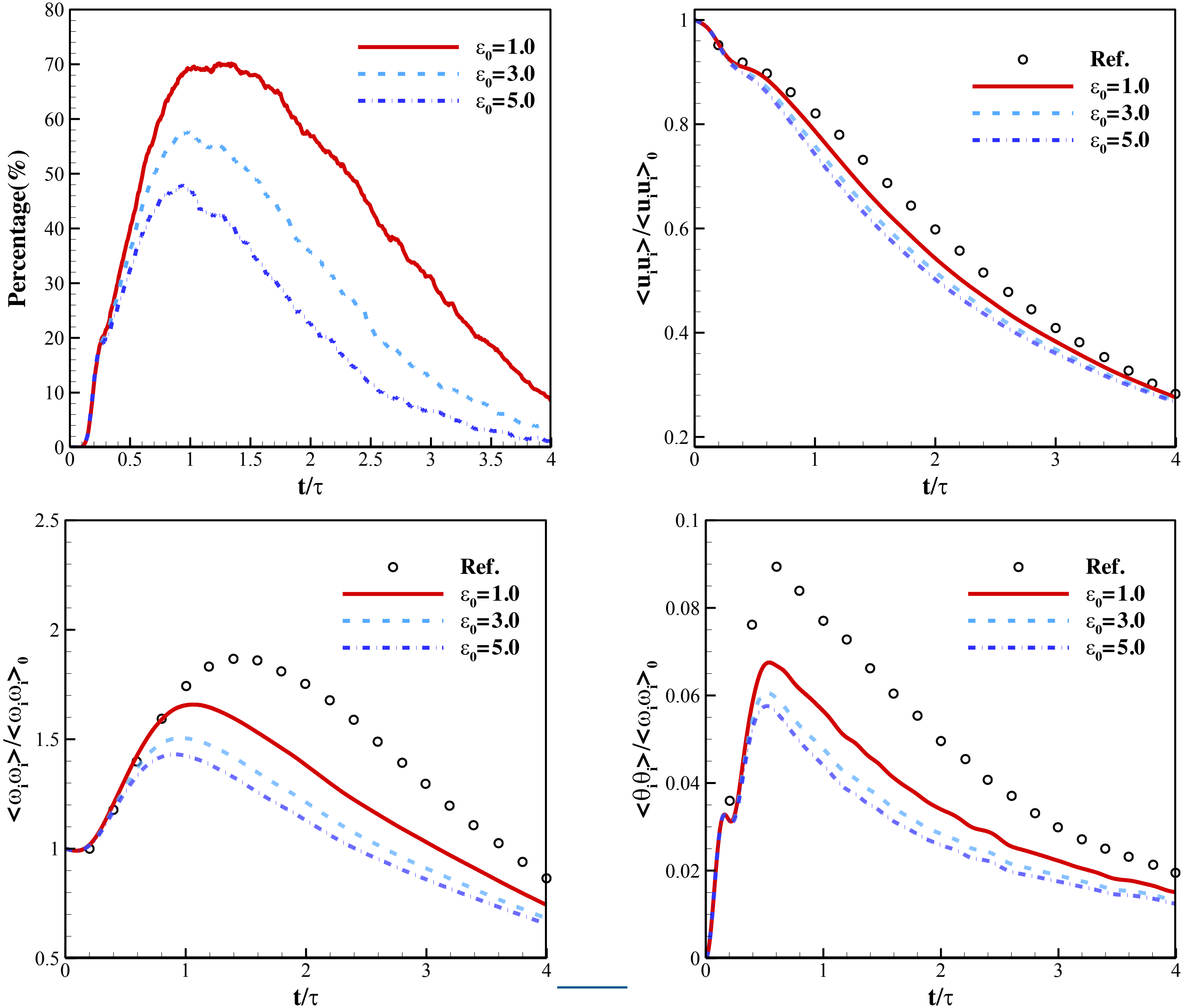
$$E(k) = u_{rms,0}^{2} 16\sqrt{\frac{2}{\pi}}\frac{k^{4}}{k_{0}^{5}}\exp\left(-\frac{2k^{2}}{k_{0}^{2}}\right)$$

- Pose demanding requirements for numerical methods, i.e. resolve multi-scale fluctuations and suppress spurious oscillations simultaneously.

*Johnsen, E., et al, “Assessment of High-Resolution Methods for Numerical Simulations of Compressible Turbulence with Shock Waves,” Journal of Computational Physics, Vol. 229, No. 4. 2010, pp. 1213–1237..

Effect of artificial viscosity on broadband accuracy

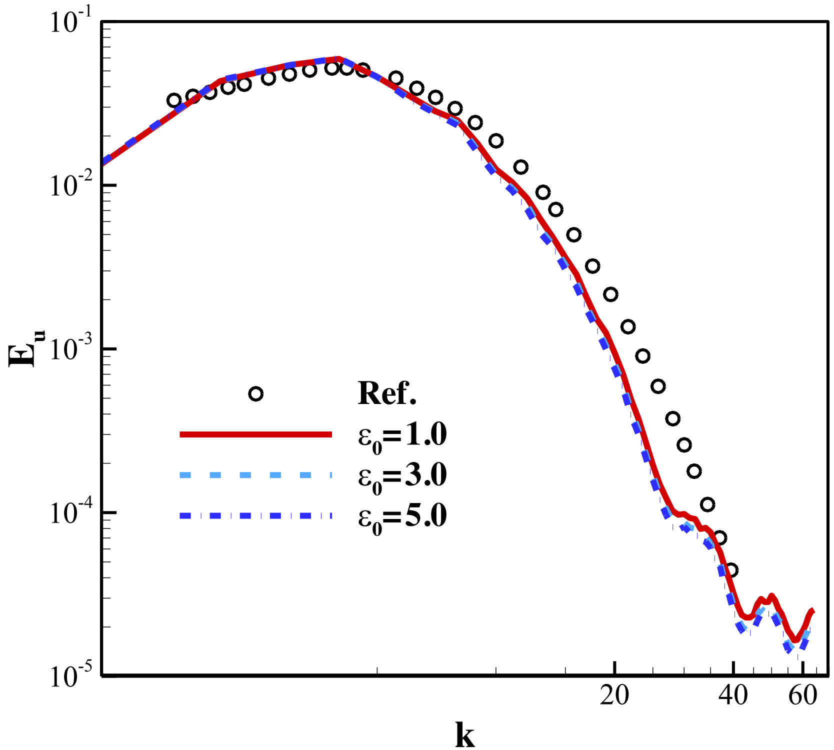
- AVDG4
- DOFs:64^3

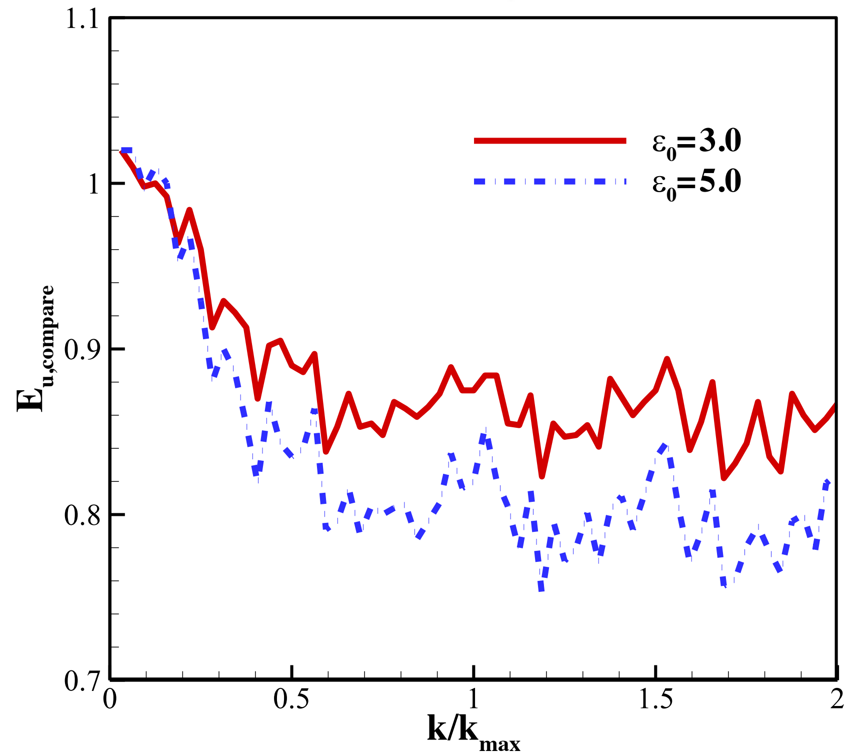


Effect of artificial viscosity on broadband accuracy

- Energy spectrum at $t = 4\tau$

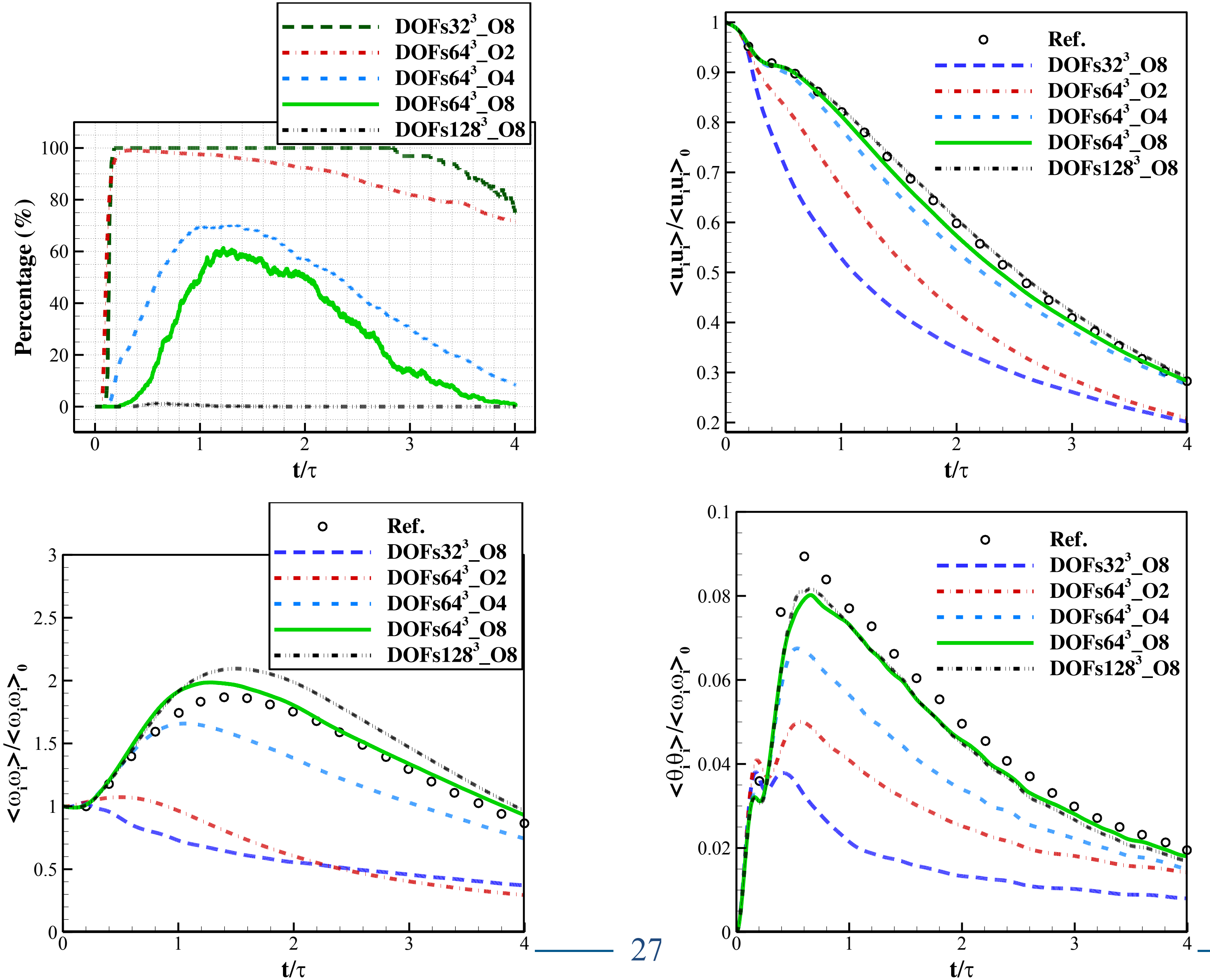
$$E_{u,\text{compare}}(k) = \frac{E_{u,\varepsilon_0}(k)}{E_{u,\varepsilon_0=1.0}(k)}$$





- Effect of artificial dissipation biased towards higher wave numbers

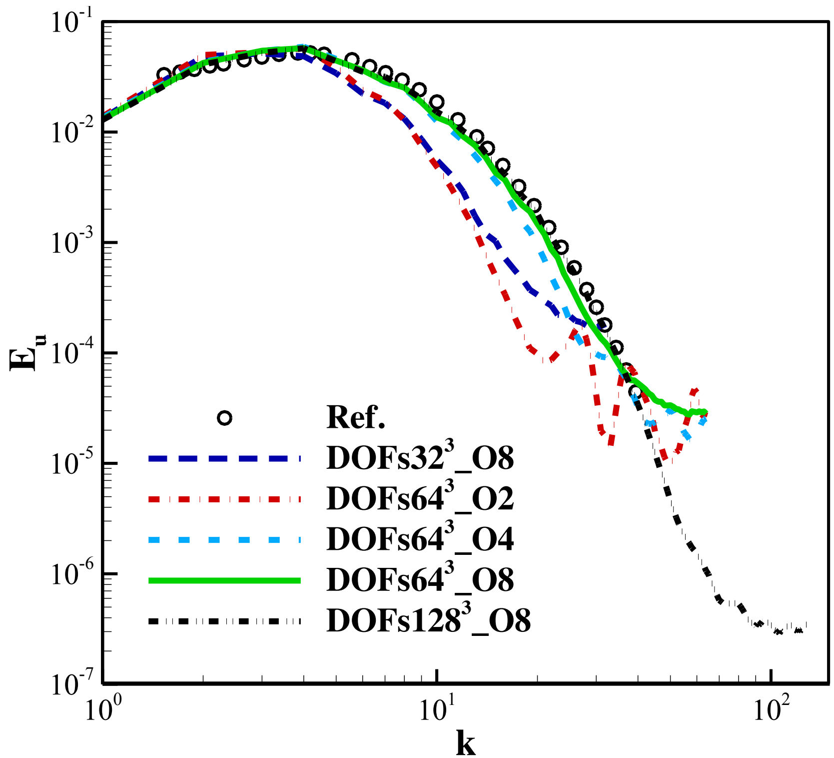
Effect of order and resolution on broadband accuracy

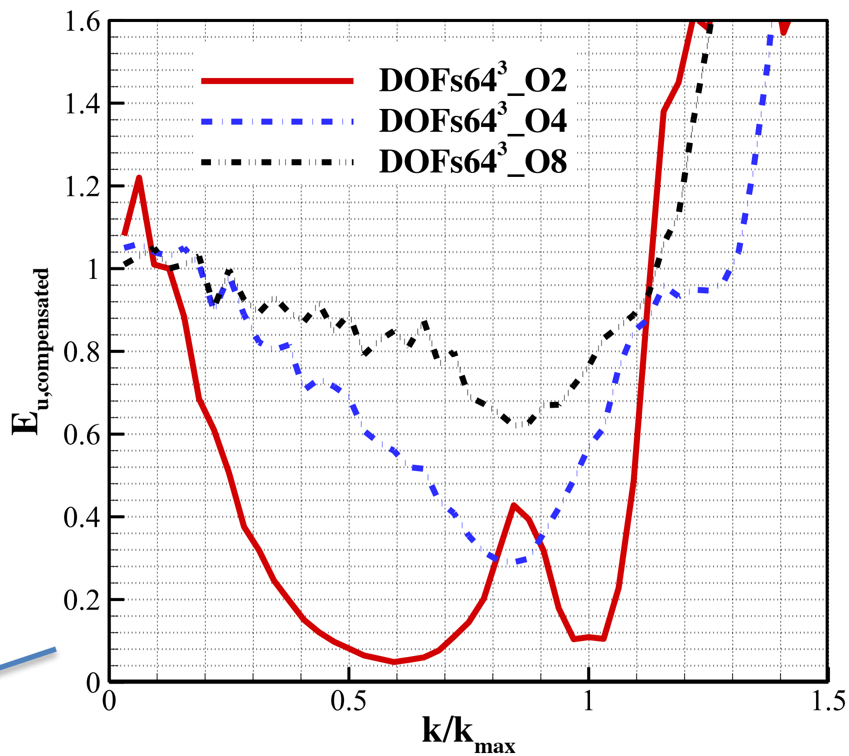


Effect of order and resolution on broadband accuracy

- Energy spectrum at $t = 4\tau$

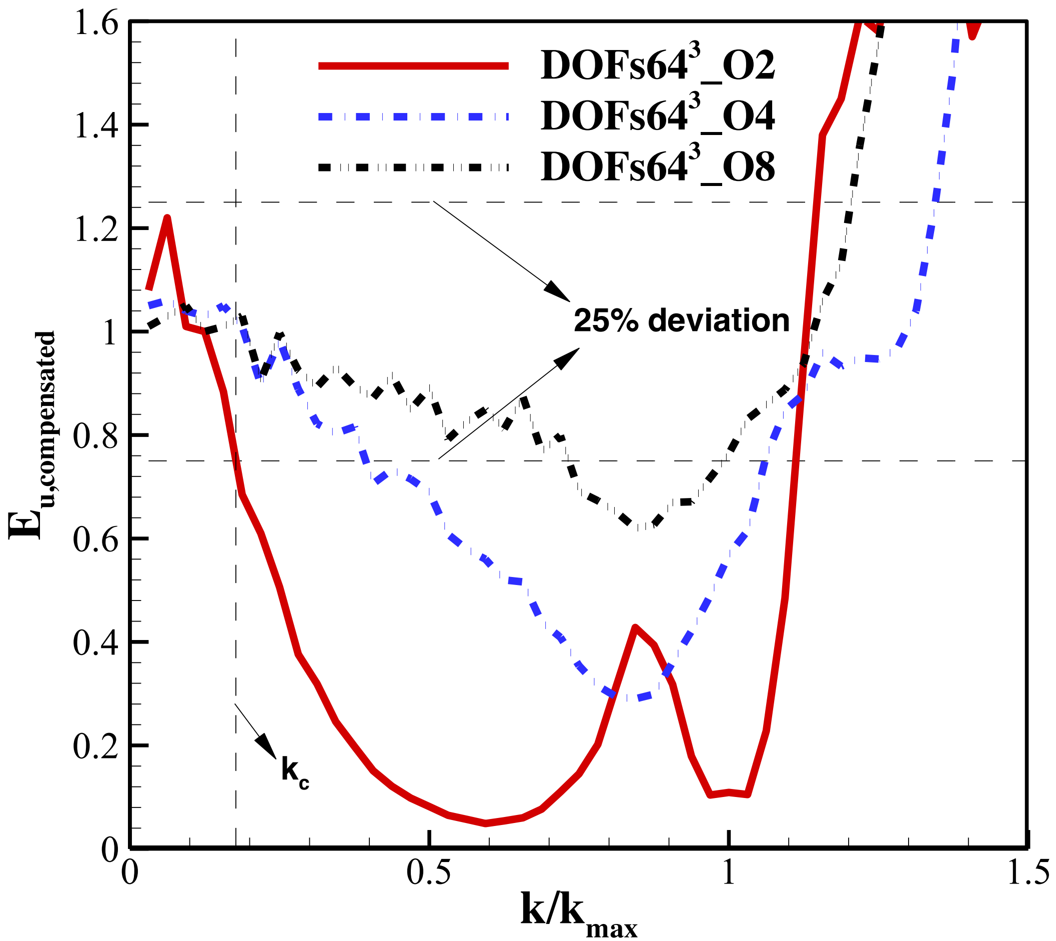
$$E_{u,\mathrm{compensated}}(k) = \frac{E_u(k)}{E_{u,\mathrm{converged}}(k)}$$





- ☐ Non-monotonic behavior attributed to the joint effect of the dissipation and the aliasing error pile-up

- Define the effective bandwidth



Effect of order and resolution on broadband accuracy

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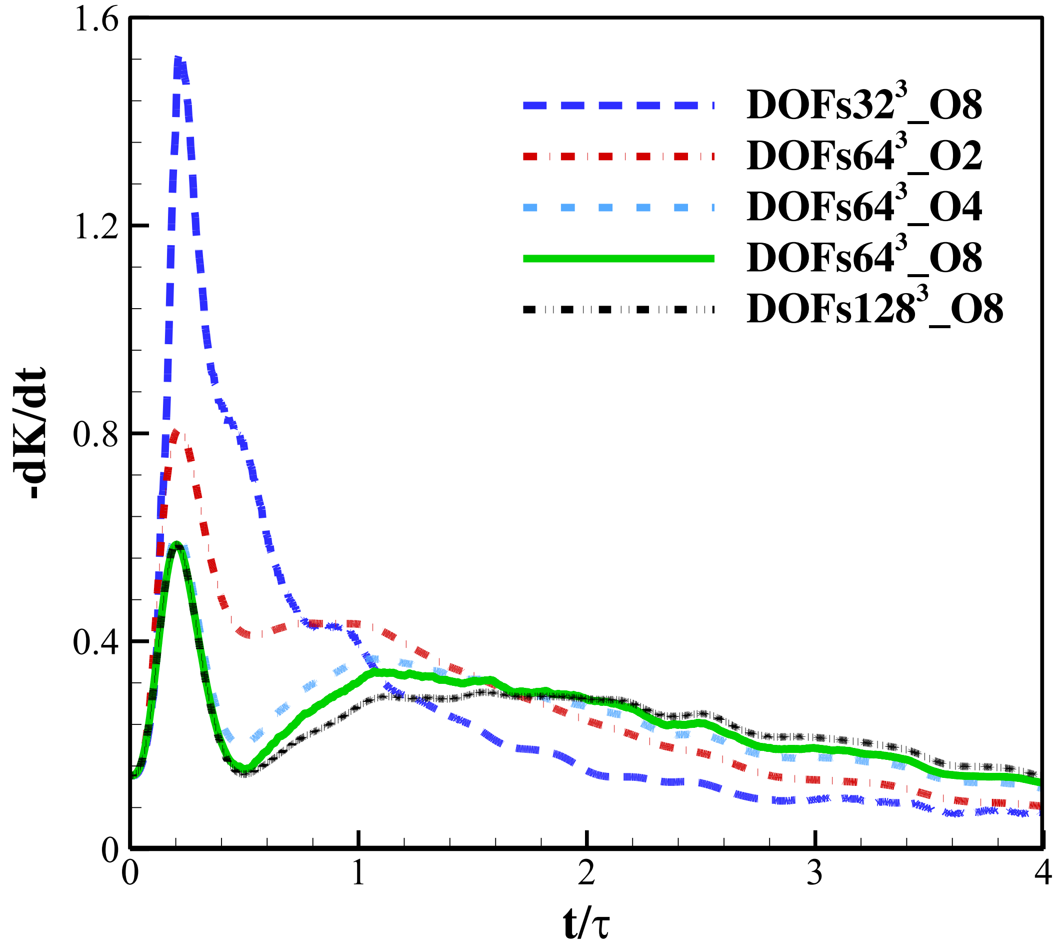
Methods	AVDG2	AVDG4	AVDG8	WENO7	Sixth-order compact scheme with artificial viscosity	WENO5/CD6
$k_{c=0.25}/k_{\max}$	0.18	0.4	0.73	0.25	0.44	0.84
$k_{c=0.4}/k_{\max}$	0.22	0.54	1.23	0.33	0.62	0.91

- Fourth order AVDG is superior to the seventh order WENO, and comparable to the sixth order compact difference scheme with an artificial viscosity model.
- Eighth order AVDG is comparable to the WENO/central difference hybrid scheme



- Discrepancy relatively small compared with each other

- Similar to the TGV case, ingredients of the total dissipation rate can be decomposed as

$$\frac{dK}{dt} = \frac{d}{dt}\left\langle \frac{1}{2}\rho u_i u_i \right\rangle = -\left(\varepsilon_{\mathrm{d}} + \varepsilon_{\mathrm{s}} + \varepsilon_{\mathrm{c}} - \langle p\theta \rangle\right)$$

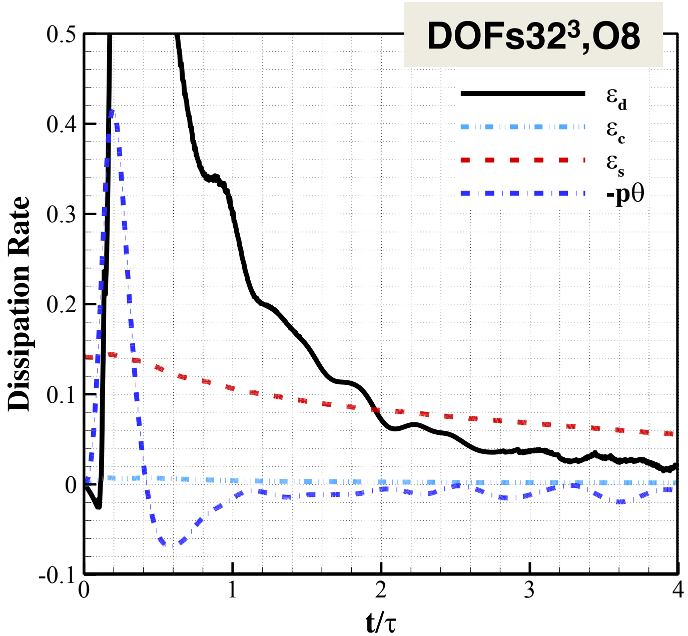
$\varepsilon_{\mathrm{s}} = \langle \mu\omega_i\omega_i \rangle$: solenoidal dissipation

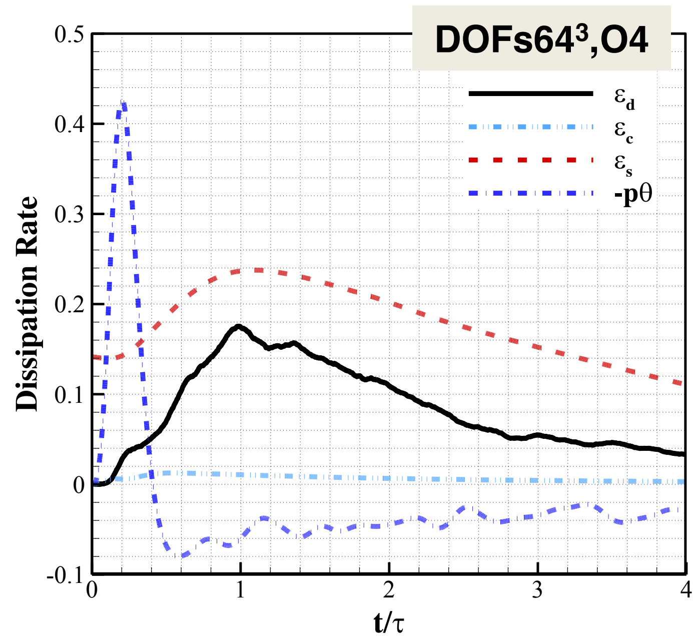
$\varepsilon_{\mathrm{c}} = \left\langle \frac{4}{3}\mu\theta^2 \right\rangle$: dilatational dissipation

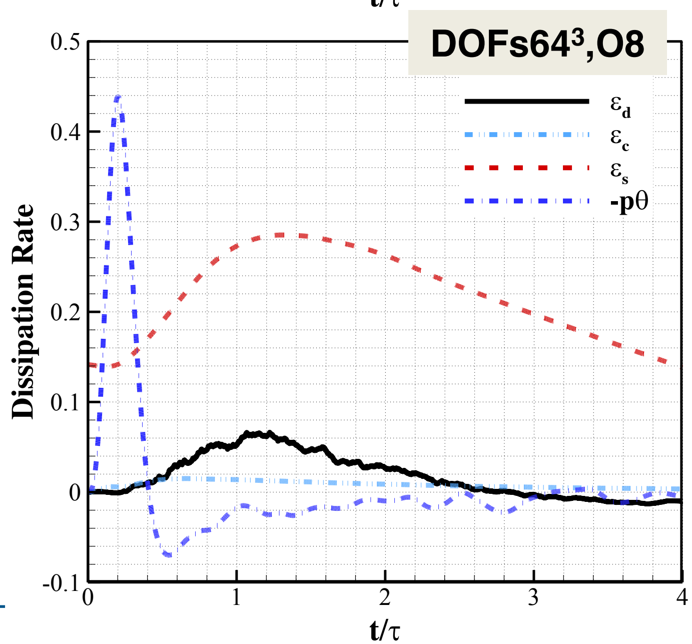
$-\langle p\theta \rangle$: pressure-dilatation transfer

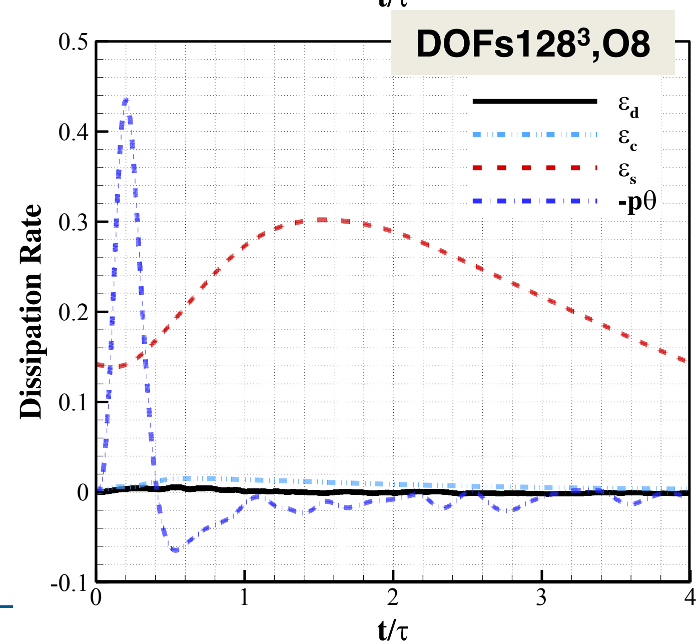
ε_{d} : numerical dissipation

Effect of order and resolution on broadband accuracy









For shock capturing

- The superior accuracy of higher order discretization for shocks is well retained with the artificial viscosity model.

For under-resolved instability(aliasing effect)

- Able to enhance stability with detrimental effects for energy
- Retain higher order accuracy property for vortical motions compared with the case with no artificial viscosity

For broadband accuracy

- The artificial viscosity model tends to affect scales of higher wavenumbers

For broadband accuracy (ctd)

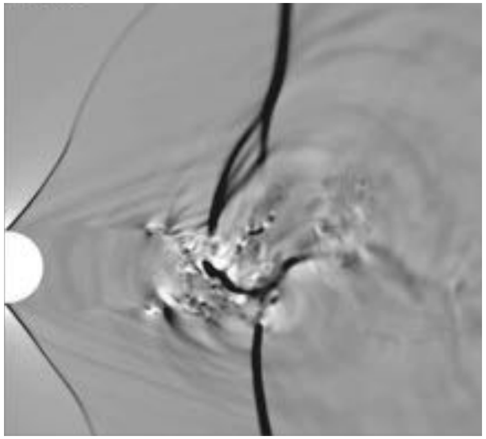
- The effective bandwidth of the fourth order AVDG method is superior to the seventh order WENO, and comparable with the sixth-order compact method with artificial viscosity
- The eighth order AVDG yields a very high value of bandwidth similar to the fifth-order WENO/sixth-order central difference hybrid method
- Numerical dissipation of the high order AVDG method is able to provide appropriate compensation for the turbulent kinetic energy on moderately coarse meshes, indicating its being a good candidate for implicit LES.

Future Plans

- Developing more accurate and robust shock capturing methods
  - ✓ Localized artificial diffusivity
  - ✓ Entropy viscosity method

- Effects of approximate Riemann flux on broadband accuracy

- Performing implicit LES for more complex flows
  - ✓ Transonic cylinder flows

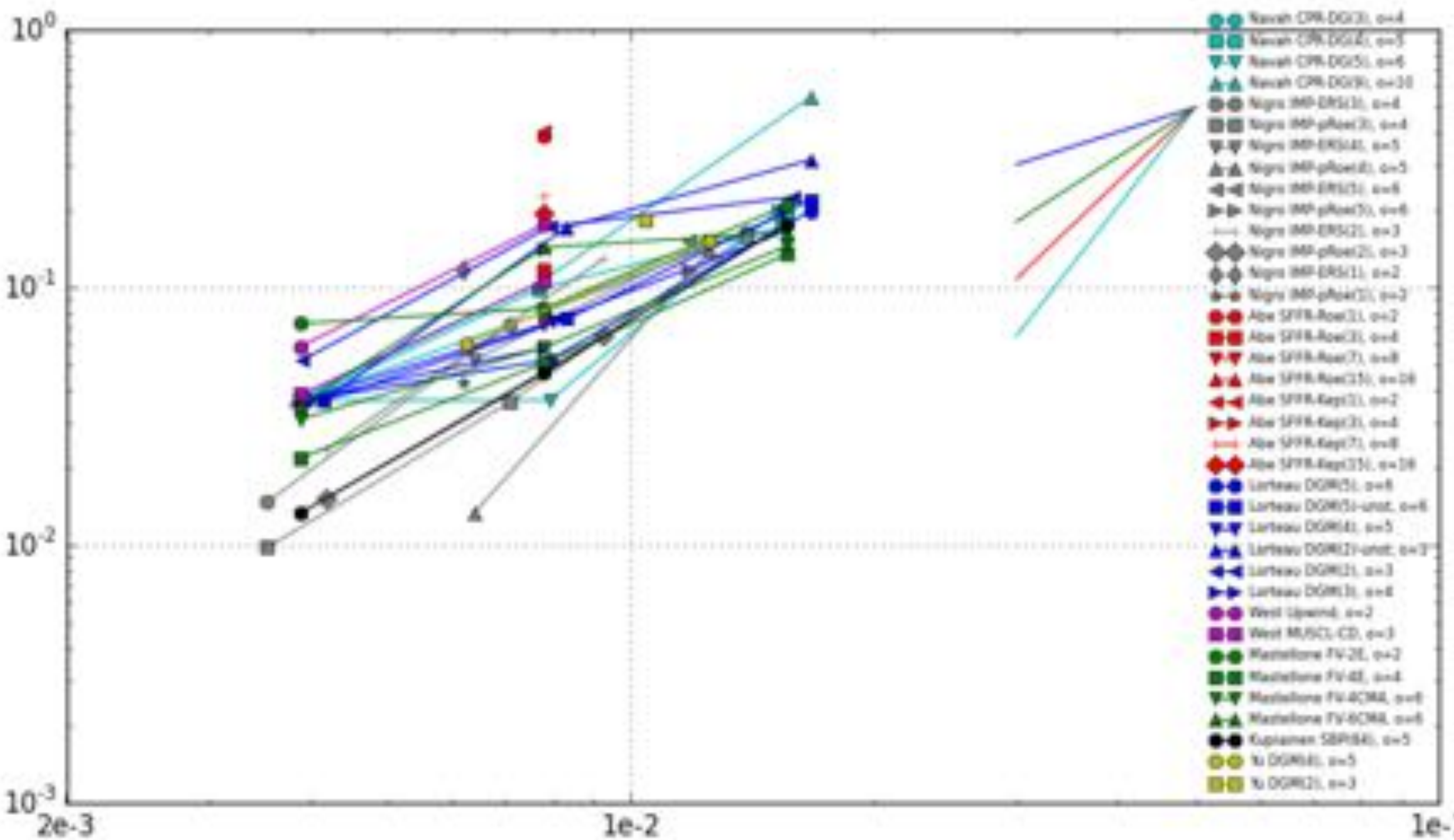


Thank you for your attention!

Questions & Suggestions?

Global results

Error on measured dissipation : resolution



Global results

Error on enstrophy dissipation : resolution