Compressible flow solver: development and applications to subsonic/transonic flows

Gianmarco Mengaldo Department of Aeronautics, Imperial College London

> LFC-UK: Development of underpinning technology for laminar flow control

Nektar++ Workshop 2015, Imperial College London

#### Supersonic / Hypersonic

Mach = u / c



Supersonic military aircraft

### 1 < Mach < 5



Hypersonic re-entry vehicle

Mach > 5



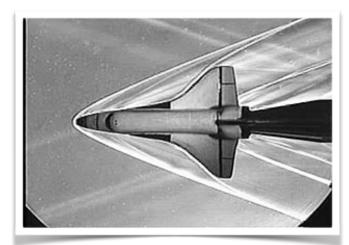
Supersonic jet exhaust. Courtesy of Parviz Moin (CTR)

### **Overview** Supersonic / Hypersonic



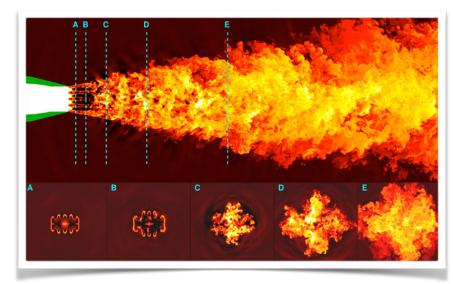
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### **Overview** Subsonic / Transonic



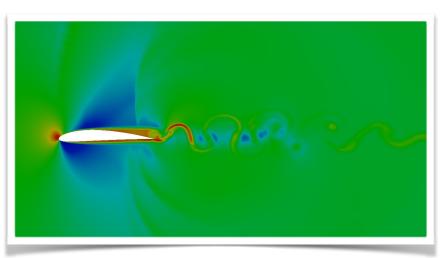
Subsonic military transport aircraft



Transonic commercial airliner

Mach < 0.8

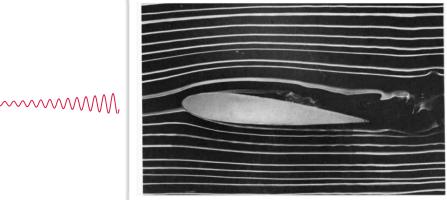
#### 0.8 < Mach < 1.2



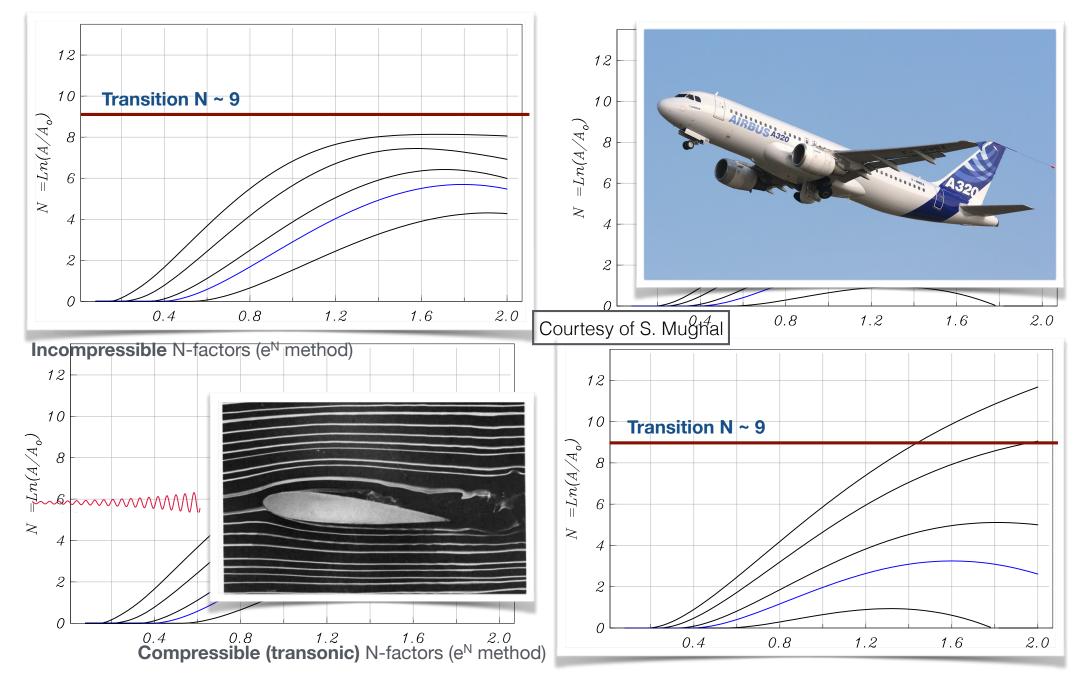
Transonic flow past an airfoil (Nektar++)

### Overview Subsonic / Transonic



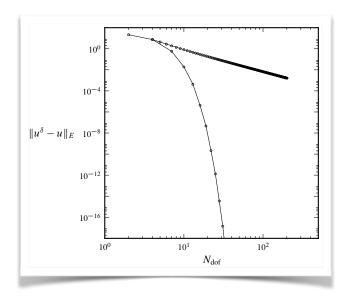


Subsonic / Transonic

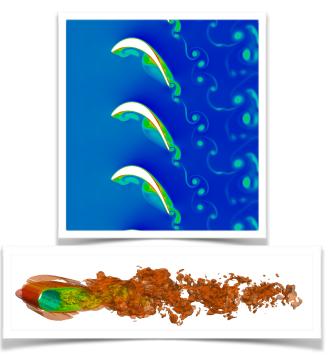


Compressible Flow Solver: Euler and Navier-Stokes equations

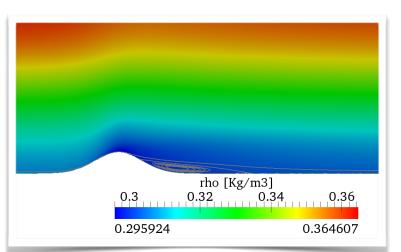
Compressible Flow Solver: Euler and Navier-Stokes equations



1. Underlying numerics (DG/FR)



2. Stabilising techniques (dealiasing)



3. Subsonic/transonic flow applications

### Outline

- 1. Underlying numerics (DG/FR)
- 2. Stabilising techniques (dealiasing)
- 3. Subsonic/transonic flow applications
- 4. Summary

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#### 1. Underlying numerics (DG/FR)

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#### Integral formulation

Discontinuous Galerkin (DG) method (Reed and Hill, 1973)

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### Integral formulation

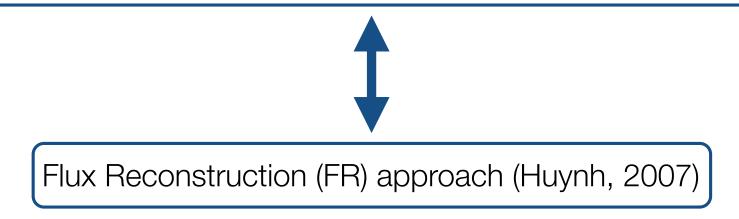
Discontinuous Galerkin (DG) method (Reed and Hill, 1973) DG-SEM (Kopriva, 2000) Nodal DG method (Hesthaven and Warburton, 2007)

Staggered grid Chebyshev multidomain (Kopriva and Kolias, 1996) Spectral difference (SD) method (Liu, Wang and Vinokular, 2006) Flux Reconstruction (FR) approach (Huynh, 2007)

Differential formulation

Discontinuous Galerkin (DG) method (Reed and Hill, 1973) DG-SEM (Kopriva, 2000)

Nodal DG method (Hesthaven and Warburton, 2007)



### Solution is discontinuous across elements

• Compressible (Euler and) Navier-Stokes equations

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathcal{H}(\mathbf{u}, \nabla \mathbf{u}) = 0$$

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$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathcal{H}(\mathbf{u}, \nabla \mathbf{u}) = 0$$
DG/FR

• Advective and diffusive flux tensors

$$\mathcal{H}(\mathbf{u}, \nabla \mathbf{u}) = \mathcal{H}^{A}(\mathbf{u}) + \mathcal{H}^{D}(\mathbf{u}, \nabla \mathbf{u})$$

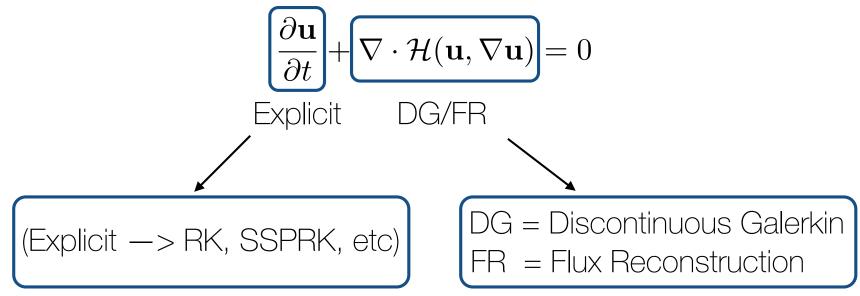
• Compressible (Euler and) Navier-Stokes equations

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathcal{H}(\mathbf{u}, \nabla \mathbf{u}) &= 0 \\ \text{Explicit} \quad \text{DG/FR} \end{aligned}$$

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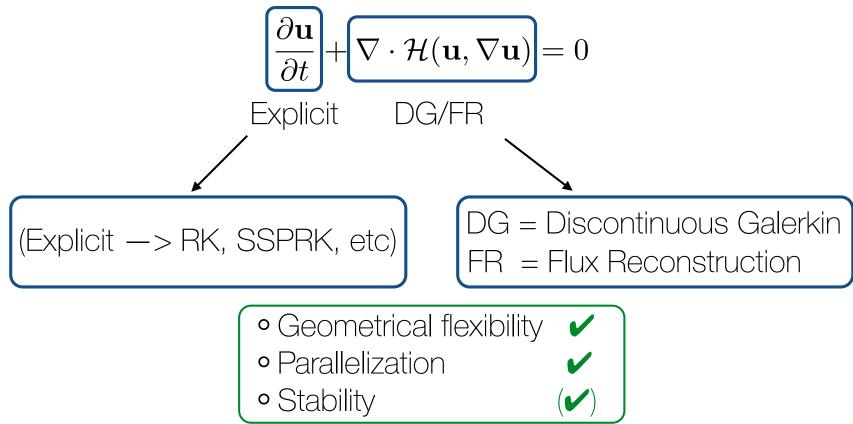
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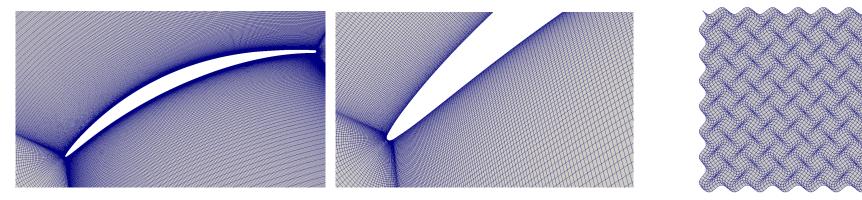
DG = Discontinuous Galerkin

FR = Flux Reconstruction

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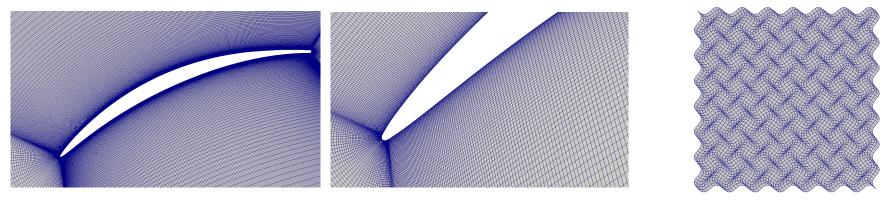
Unstructured (deformed/curved) elements



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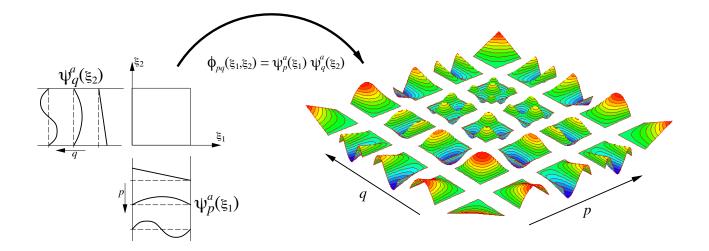
#### Map each element into a reference element

$$\overleftarrow{\boldsymbol{\xi}} = \Theta(\boldsymbol{x})$$

DG = Discontinuous Galerkin

FR = Flux Reconstruction

Define an expansion basis for each element



Up to now they are identical ...

DG = Discontinuous Galerkin

Weak formulation

FR = Flux Reconstruction

#### **Strong formulation**

DG = Discontinuous Galerkin

FR = Flux Reconstruction

**Weak formulation** 

**Strong formulation** 

**Encapsulates DG among others<sup>3</sup>** 

<sup>3</sup>P.E. Vincent, P. Castonguay and A. Jameson Insights from Von Neumann analysis of high order flux reconstruction schemes Journal of Computational Physics, 2011

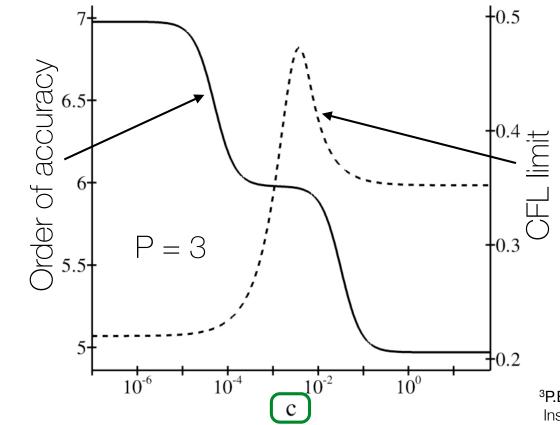
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#### **Weak formulation**

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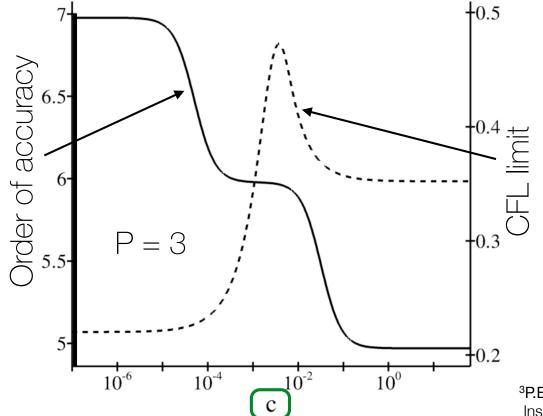
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c = 0 FR<sub>DG</sub>

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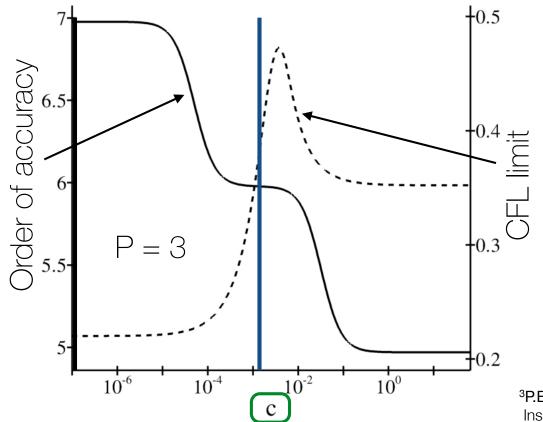
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 $c = 0 \quad \text{FR}_{\text{DG}}$   $c = \frac{2(P+1)}{(2P+1)P(a_P P!)^2} \quad \text{FR}_{\text{g2}}$ 

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 FR<sub>DG</sub>

DG with exact mass matrix (EMM)

-> DGsem

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DG with exact mass matrix (EMM)  $-> DG_{SEM}$ 

#### True for a linear problem in 1D

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#### **True for a linear problem in 1D**

We extended the proof to any nonlinear equation<sup>1</sup> We extended the proof to any deformed/curved mesh<sup>1</sup>

<sup>1</sup>D. De Grazia, G. Mengaldo, D. Moxey, P.E. Vincent and S.J. Sherwin Connections between the discontinuous Galerkin method and high-order flux reconstruction schemes INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS, 2014, Volume 75, Issue 12

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Performance & Stability ()?

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Performance & Stability ()?

Inexact integration

Collocation-based

Exact integration

#### Outline

#### 1. Underlying numerics (DG/FR)

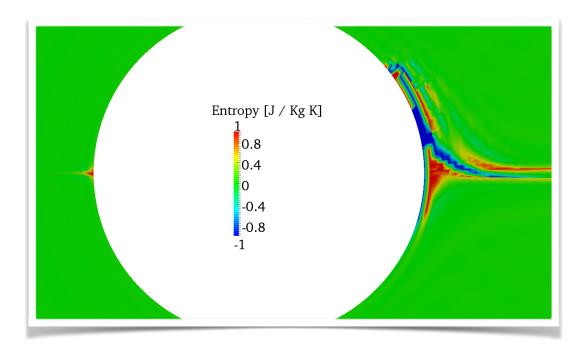
2. Stabilising techniques (dealiasing)

3. Subsonic/transonic flow applications

4. Summary

Under- or marginal-resolution High-Reynolds number flows Inexact integration of the nonlinearities

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Unstable simulation due to under-resolution errors and inexact integration

- Consistent integration of the nonlinearities
- Spectral Vanishing Viscosity (SVV) + others

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- Consistent integration of the nonlinearities
- Spectral Vanishing Viscosity (SVV) + others
- More stable forms of the governing equations
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   ...
- Optimal choice of the quadrature points

• Consistent integration of the nonlinearities

+2

• Spectral Vanishing Viscosity (SVV)

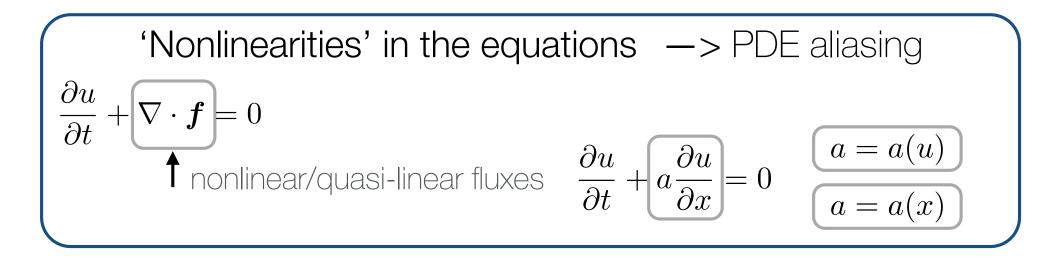
• Consistent integration of the nonlinearities<sup>3</sup>

#### Stabilising techniques Aliasing sources

**'Nonlinearities' in the equations** —> PDE aliasing

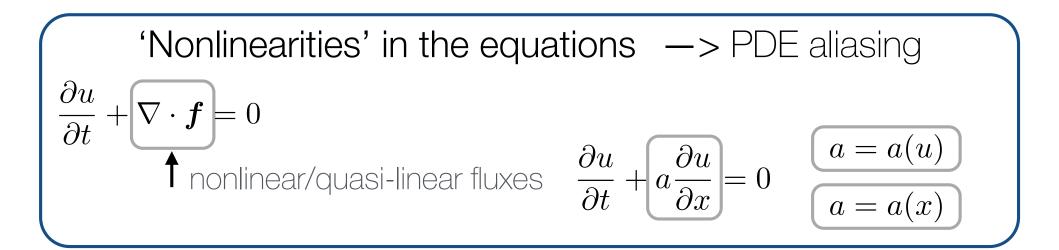
**'Nonlinearities' in the geometry** —> Geometrical aliasing

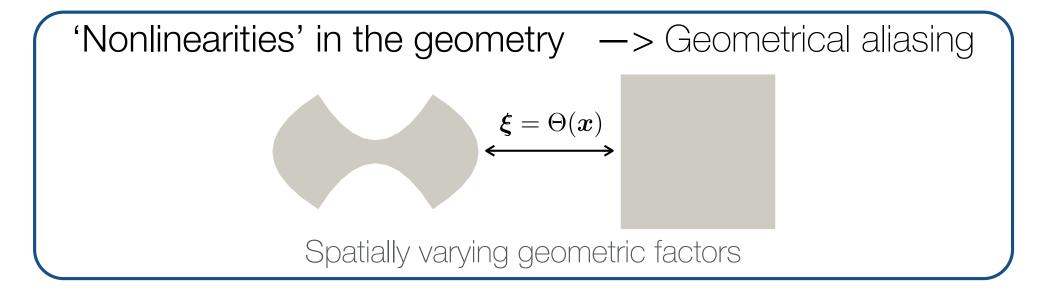
Aliasing sources



#### 'Nonlinearities' in the geometry —> Geometrical aliasing

Aliasing sources





Local 
$$\mathsf{DG} \quad \frac{\mathrm{d}u}{\mathrm{dt}} = \mathsf{M}^{-1} \Big|_{Q_P} \left( \mathcal{L}_V \Big|_{Q_P} + \mathcal{L}_I \Big|_{Q_P} + \mathcal{N}_V \Big|_{Q_V} + \mathcal{N}_I \Big|_{Q_I} \right)$$
$$\mathsf{FR} \quad \frac{\mathrm{d}u}{\mathrm{dt}} = \mathcal{L}_V \Big|_{Q_P} + \mathcal{L}_I \Big|_{Q_P} + \mathcal{N}_V \Big|_{Q_V} + \mathcal{N}_I \Big|_{Q_I}$$

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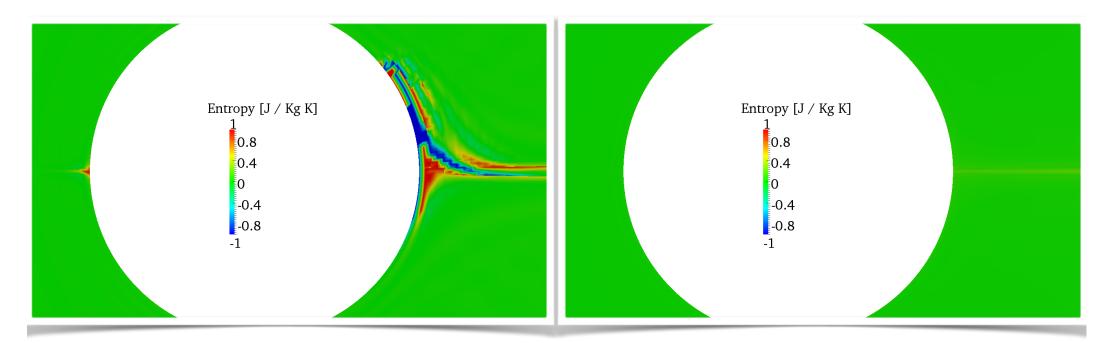
Global 
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Flow applications

Compressible Euler equations

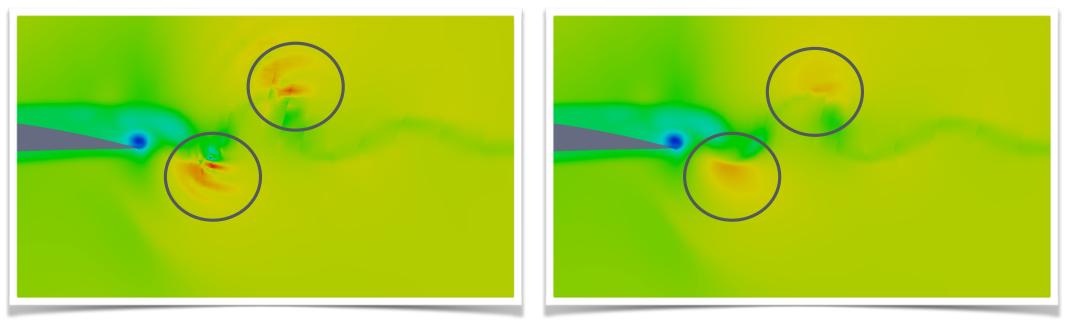


Unstable simulation due to underresolution and under-integration errors

Stable simulation with Local dealiasing technique

#### Stabilising techniques Flow applications

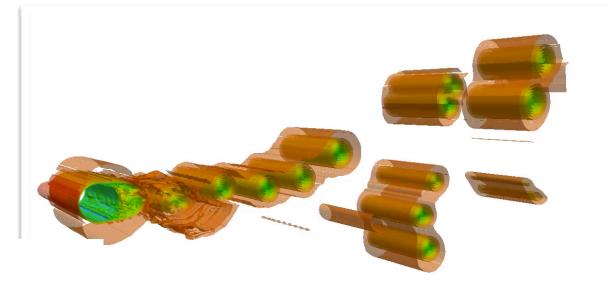
Compressible Navier-Stokes equations - NACA 4412



Unstable simulation due to underresolution and under-integration errors

Stable simulation with Local dealiasing technique

#### Flow applications



Cylinder - Density Mach = 0.2, Re = 3900 Local dealiasing

T106C blade - Temperature Mach = 0.2, Re = 200 Local dealiasing



# Summary Summary

- Consistent integration improves robustness
- Local/global dealiasing techniques can be equally applied to DG and FR
- Local approach enhance efficiency
- Additional details<sup>3</sup>

#### Outline

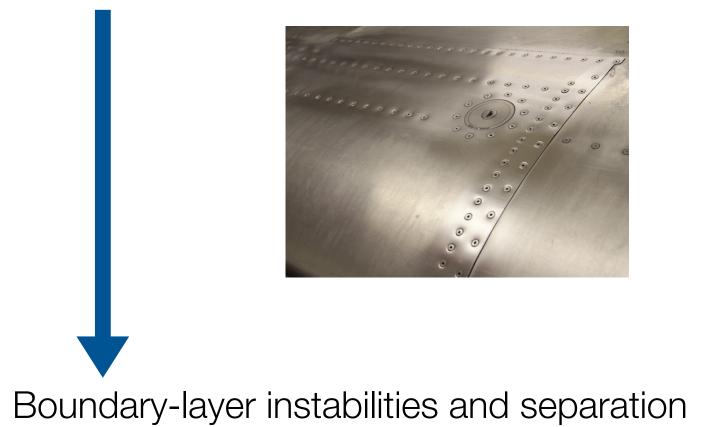
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Roughness elements / steps / gaps / imperfections are a common feature on wings



Boundary-layer instabilities and separation

Reduced models (triple-deck theory)

Numerical solution of the governing equations (DNS/LES)

Wind tunnel experiments

LFC-UK

Flight experiments



Boundary-layer instabilities and separation

Focus of this work

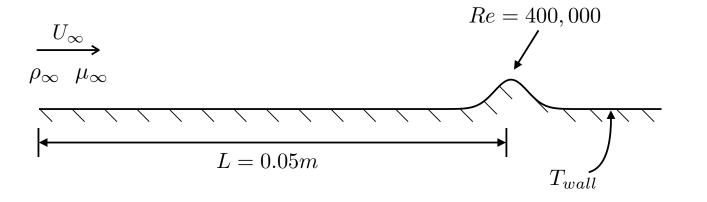
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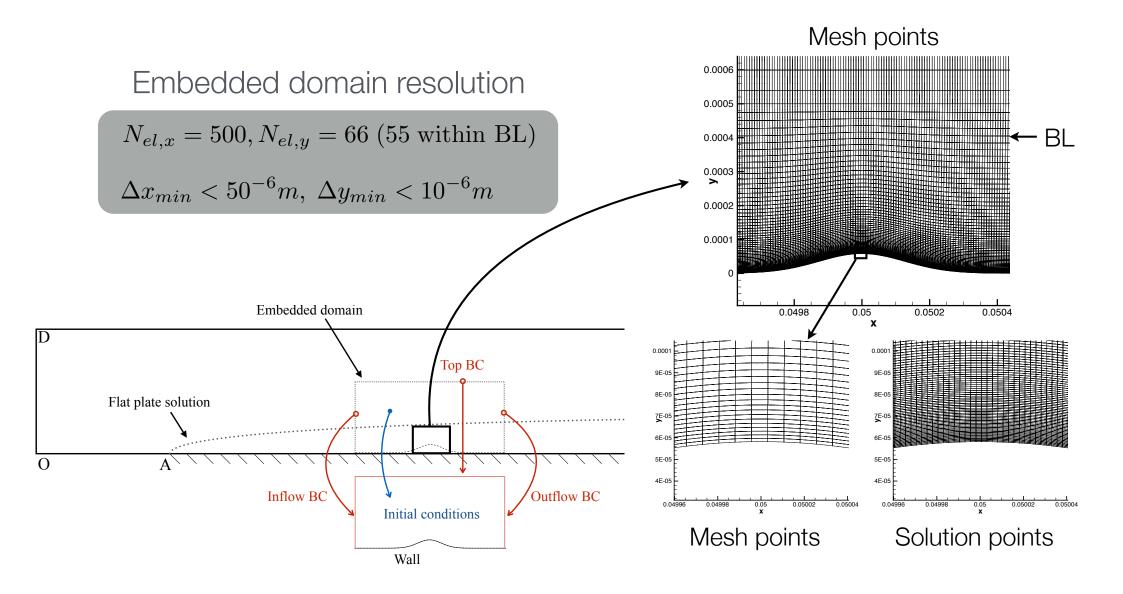
LFC-UK

Flight experiments

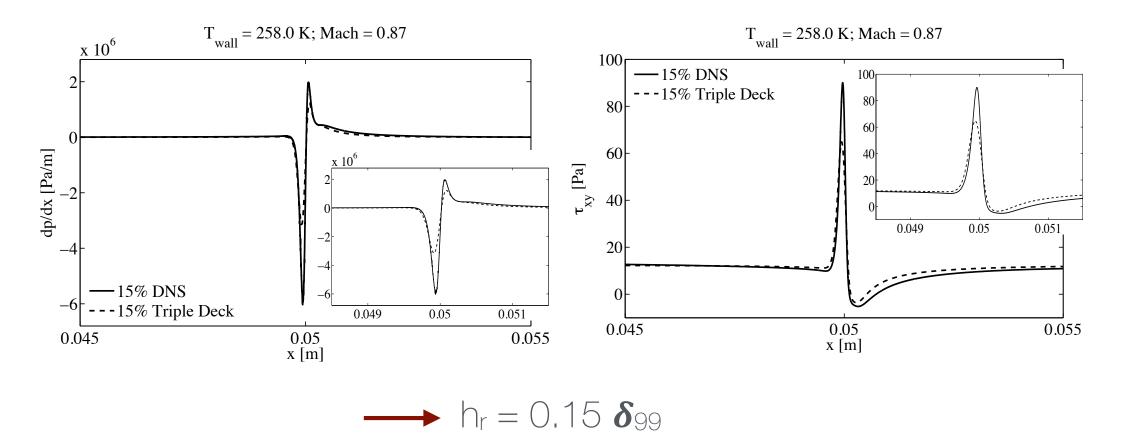


#### Idealised small imperfections at leading-edge

	$M = \frac{U_{\infty}}{c_{\infty}}$	$T_{wall}$	$h_r$	$Re = \frac{\rho U_{\infty}L}{\mu}$	$Pr = \frac{C_p \mu}{\kappa}$
Case 1	0.50	216.29 K	$[0.05, 0.10, 0.15]\delta_{99}$	4x10 <sup>5</sup>	0.72
Case 2	0.87	258.0 K	$[0.05, 0.10, 0.15]\delta_{99}$	4x10 <sup>5</sup>	0.72



<sup>4</sup>G. Mengaldo, M. Kravtsova, A.I. Ruban, and S.J. Sherwin Triple-deck and direct numerical simulation analyses of high-speed subsonic flows past a roughness element Journal of Fluid Mechanics, 2015, Volume 774

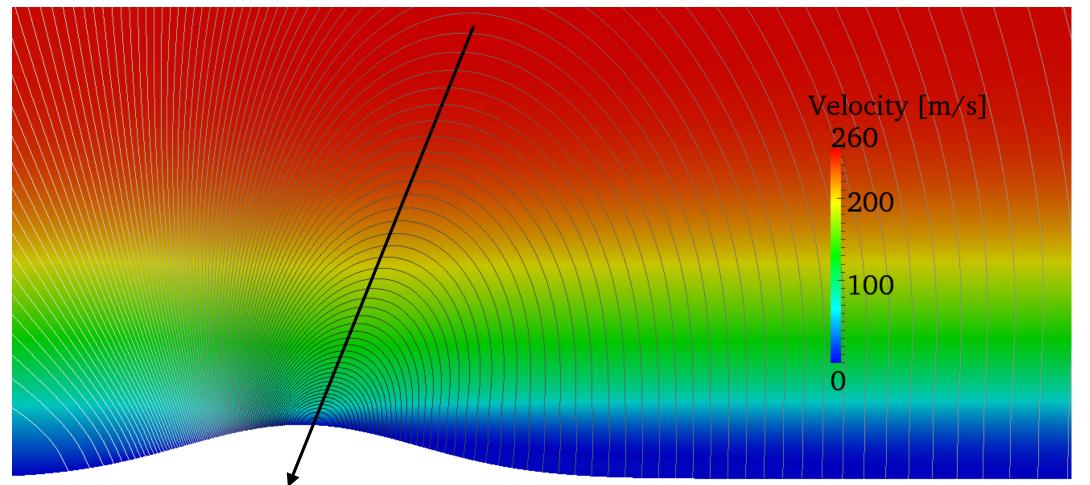


• Differences increase with hump height

#### <sup>4</sup>G. Mengaldo, M. Kravtsova, A.I. Ruban, and S.J. Sherwin

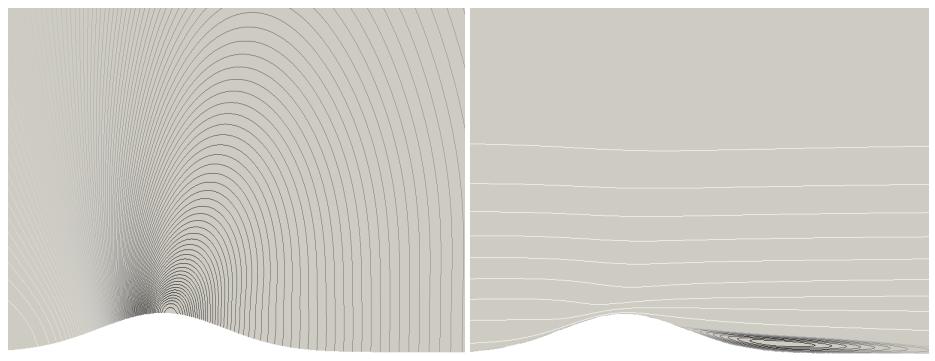
Triple-deck and direct numerical simulation analyses of high-speed subsonic flows past a roughness element Journal of Fluid Mechanics, 2015, Volume 774

Grey lines = iso-contours of pressure



Wall-normal pressure gradient

Iso-contours of pressure Iso-contours of velocity

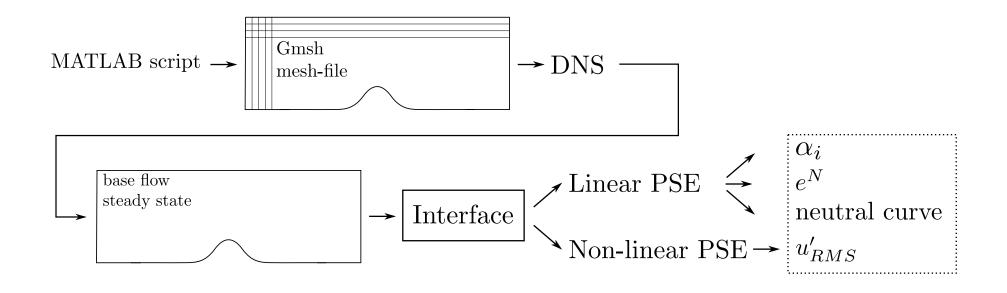


Wall-normal pressure gradient

- sharper and higher peaks in DNS
- different flow acceleration/deceleration
- noticeable difference in the reattachment point

Framework for linear/nonlinear PSE & LST on complex geometries

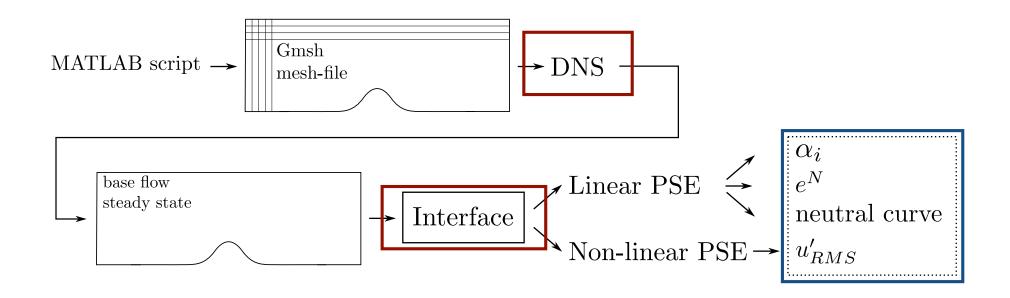
Framework for linear/nonlinear PSE & LST on complex geometries



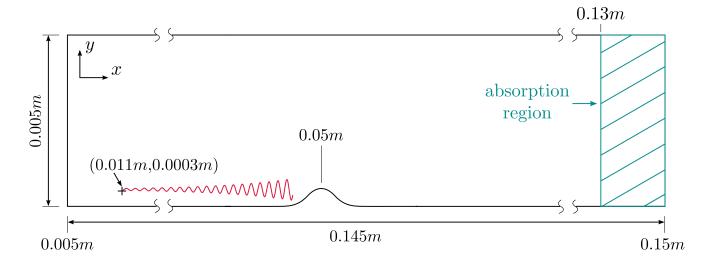
Framework for linear/nonlinear PSE & LST on complex geometries

Nektar++ to obtain high-fidelity DNS data

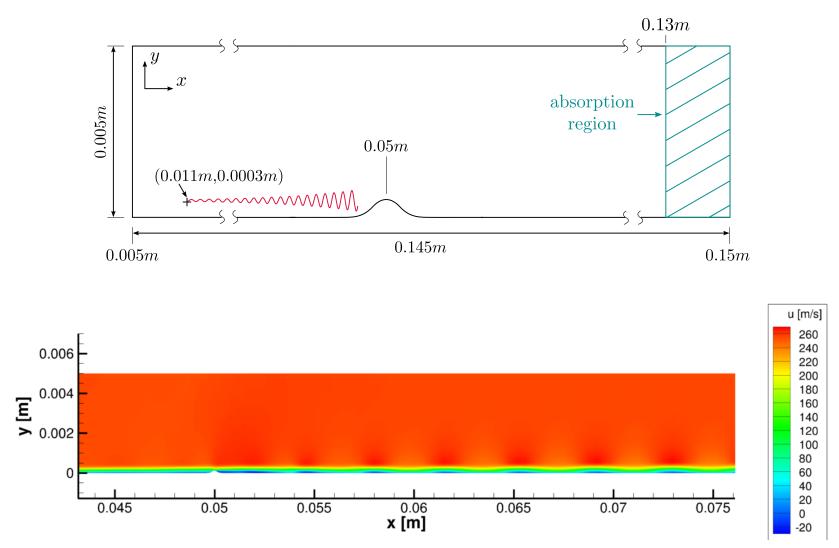
Interface tool for existing linear/nonlinear PSE & LST codes

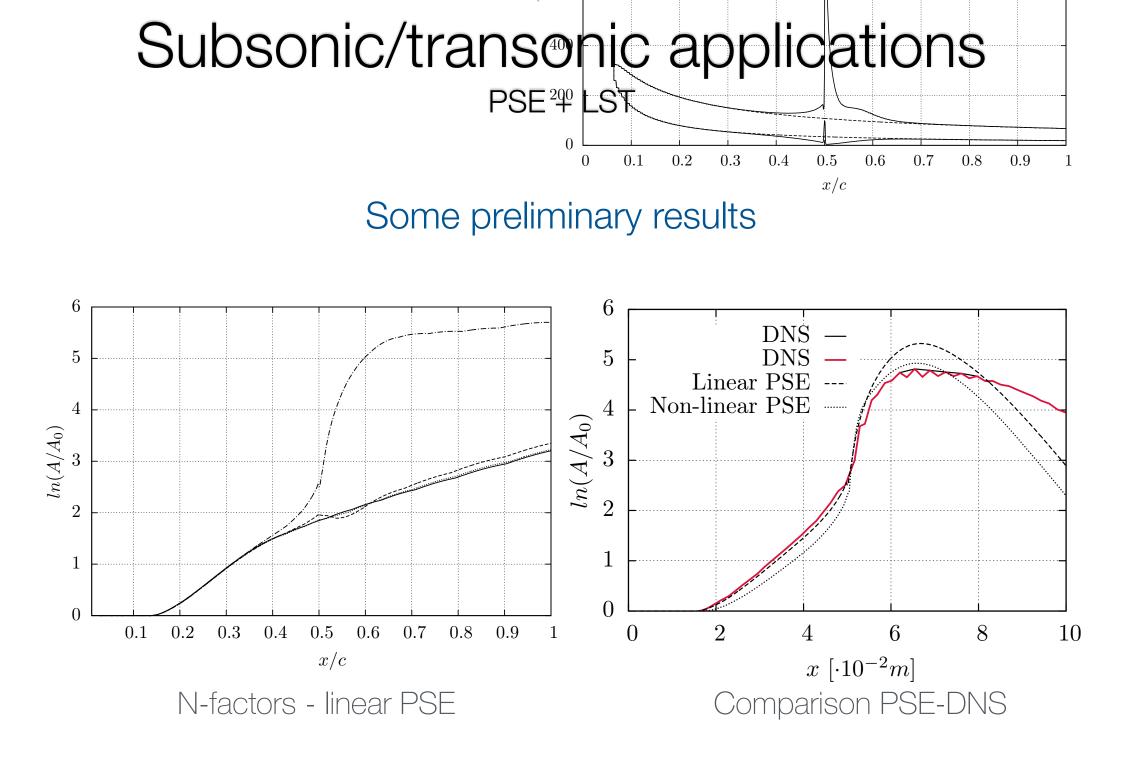


#### Flow configuration



#### Flow configuration





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1. Underlying numerics (DG/FR)

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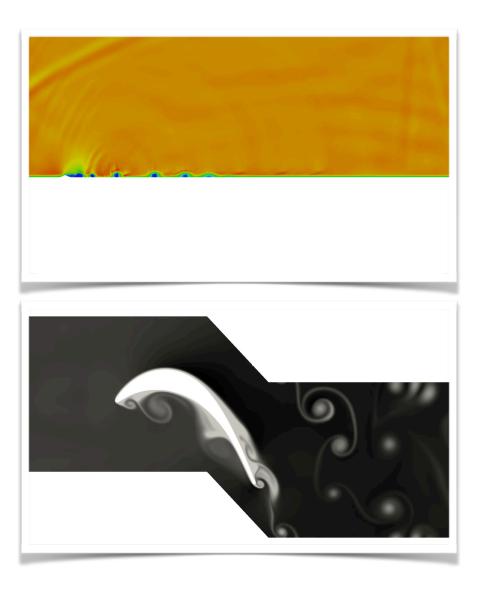
#### Summary

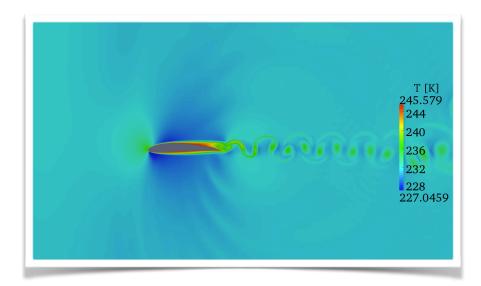
- Robust (and efficient) numerical framework for compressible flows
   Euler equations
  - Navier-Stokes
- Implementation of both DG and FR (FR for 1D and 2D quad only)
- Implementation of both local and global dealiasing techniques
- Successfully applied to high-Reynolds number aeronautical problems
- Boundary conditions? —>

#### G. Mengaldo et al.

A Guide to the Implementation of Boundary Conditions in Compact High-Order Methods for Compressible Aerodynamics, 2014, AIAA Aviation

#### Thank you!







Shock-capturing / P-adaption / adjoint formulation are also included! —> Next presentation by Dirk Ekelschot!