Shear stress distributions and Tollmien-Schlichting waves in boundary layers

Simulated by Nektar++

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Outline

1. Simulated results
   - Validations & some coding work
   - Cases of flat plate & smooth wavy walls
2. Ongoing works

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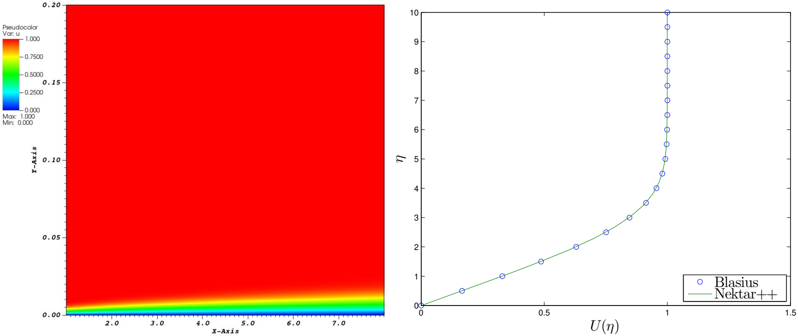
Blasius boundary layer

Basic parameters (20/01/2013):

$Re_{x=1} = 400\,000$. $\Omega = [1, 8] \times [0, 0.2]$. Mesh numbers: 3899 Triangles & 2232 Quadrangles. Modes: 7.



Derivative: $\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0.33196516$ (at $x = 5$) (=0.33197665 at $x = 7$)

Kowarth reports $f''(0)$ = 0.332057.

Implement sponge layer

The sponge layer for both nonlinear and linearized NSEs:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \nu \Delta u + (u \cdot \nabla) u + \nabla p & = -\sigma(x, y)(u - U) + f, \quad \text{in } \Omega \\ \nabla \cdot u & = 0, \quad \text{in } \Omega \end{cases}$$

$$\begin{cases} \dfrac{\partial u'}{\partial t} - \nu \Delta u' + (U \cdot \nabla) u' + (u' \cdot \nabla) U + \nabla p & = -\sigma(x, y) u' + f, \quad \text{in } \Omega \\ \nabla \cdot u' & = 0, \quad \text{in } \Omega \end{cases}$$

where $\sigma(x, y)$ is a spatial dependent absorbing strength and u' is a perturbation of the flow around base flow. By the above definition, the sponge layer can be used for both DNS and linearized NSEs.

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Schematic geometry with wavy wall (07/02/1013)

Figure: Basic wavy wall geometry

Simulation parameters:

Table: Parameters: $\Omega = [1,4] \times [0,0.2]$. $x_s = 1.5$ is the start position of wavy walls. Ref. velocity u=1. $Re = 1/\nu = 400000$. h (2 times of wave amplitude) and λ denote the wavy wall depth and wavelength of wavy walls.

C	$Re_{x_{\text{Inlet}}}$	δ_{inlet}	h	λ	k	$4.91 x_s Re_{x_s}{}^{-1/2}$	$4.91 x_s Re_{x_s}{}^{-5/8}$
*	400000	0.0078	NA	NA	NA	0.0095	0.0018
1	\	\	0.001	0.2	1	\	\
2	\	\	0.002	0.2	1	\	\
3	\	\	0.003	0.2	1	\	\

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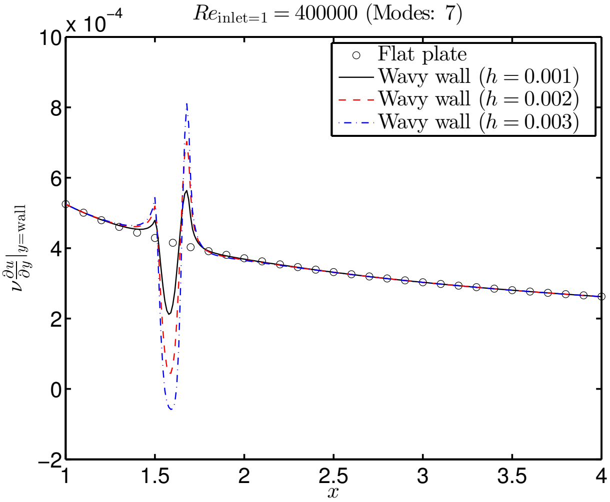
Wall shear stress distributions



Tollmien-Schlichting waves

Exciters: The excite center position is at $x_{st} = 1.25$ and the shape of the exciter $A(x) = \epsilon A_m(x)\sin(\omega t)$ is defined by

$$A_m = a_1 \left(\frac{x - x_1}{x_{st} - x_1}\right)^5 - a_2 \left(\frac{x - x_1}{x_{st} - x_1}\right)^4 + a_3 \left(\frac{x - x_1}{x_{st} - x_1}\right)^3, \ x_1 \leq x \leq x_{st},$$

and

$$A_m = -a_1 \left(\frac{x_2 - x}{x_2 - x_{st}}\right)^5 + a_2 \left(\frac{x_2 - x}{x_2 - x_{st}}\right)^4 - a_3 \left(\frac{x_2 - x}{x_2 - x_{st}}\right)^3, \ x_{st} \leq x \leq x_2,$$

where $x_{st} = \frac{x_1 + x_2}{2}$, $a_1 = 15.1875$, $a_2 = 35.4375$, $a_3 = 20.25$. $\epsilon = 0.0001$ is the amplitude of the exciter. $x_1 = 1.2$ and $x_2 = 1.3$. For all simulations in this part, the sponge regions are used in the range $x \in [3.8, 4]$. The exciter dimensionless frequency $\omega = 0.0689$.

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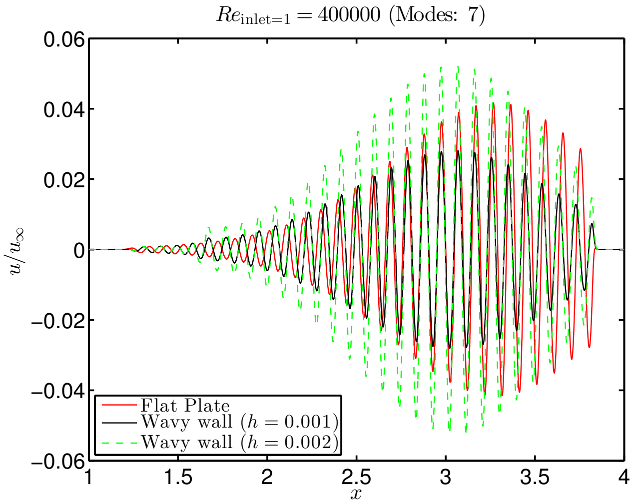
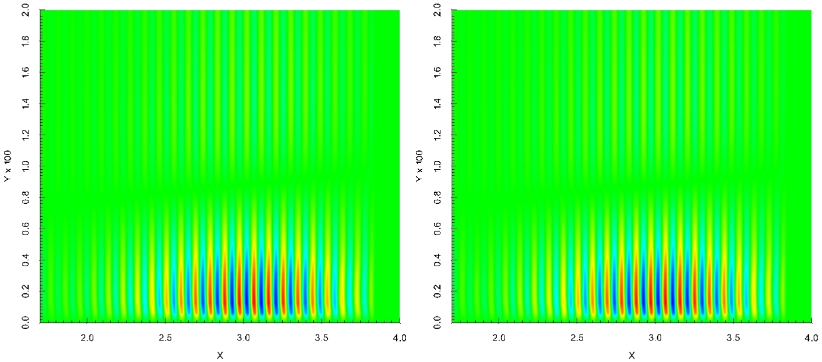
Tollmien-Schlichting waves

Comparisons of T-S wavs (at y=0.001, $\epsilon = 0.0001$)



Tollmien-Schlichting waves



(a) $h = 0.001$ (b)$h = 0.002$

Figure: Horizontal velocity u for wavy walls.

- Looking for scaling laws;
- Theoretical analysis on the reduction of T-S waves;
- 2.5D simulations;
- ......

Enjoy a movie of T-S waves' evolution with a single wave front.