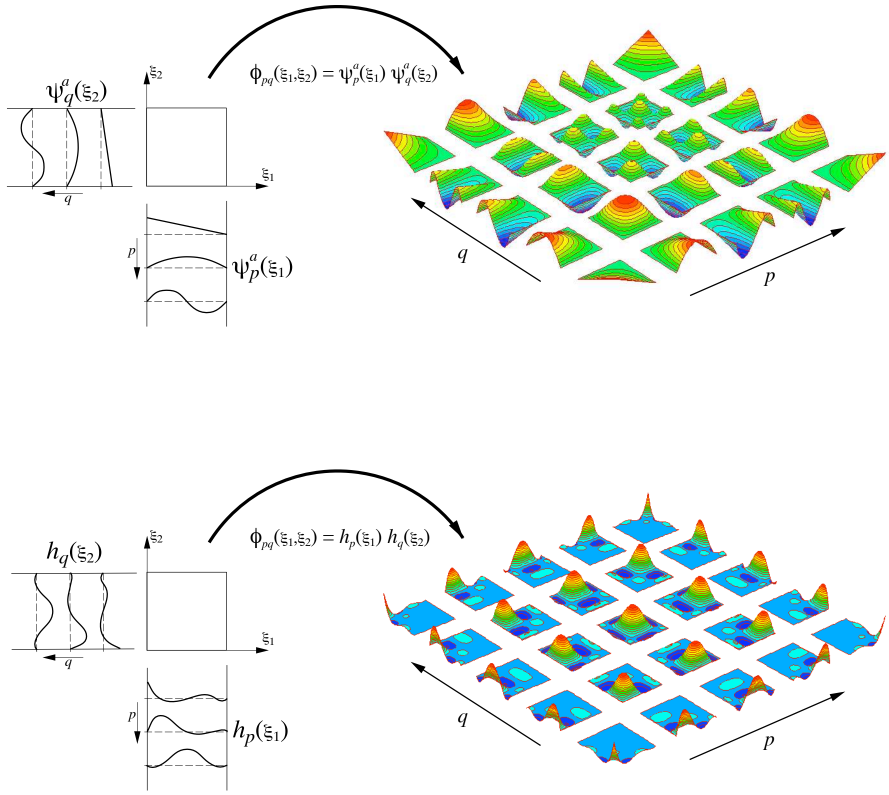

Figure 17: Construction of a two-dimensional expansion basis from the tensor product of two one-dimensional expansions of order $P = 4$. A modal expansion (top) and a nodal expansion (bottom) are shown.

expansion. Since a large part of the efficiency of the quadrilateral expansion (particularly at larger polynomial orders) arises from the tensor product construction, we would like to use a similar procedure to construct expansions within the triangular domains. Therefore, to extend the tensor product expansion to simplex regions such as a triangle we need to generalise the tensor product expansion concept, which can be achieved by using a *collapsed coordinate system*.

4.1.3 Collapsed coordinate system

In this section we will focus on 2D expansions defined on the standard triangle $\mathcal{T}_{st}$, defined as

$$\mathcal{T}_{st} = \{(\xi_1\,,\xi_2)| -1 \leq \xi_1\,,\xi_2\,;\xi_1+\xi_2 \leq 0\}\ .$$

In the quadrilateral expansions discussed in section 4.1.1 we generated a multidimensional expansion by forming a tensor product of one-dimensional expansions based on a Cartesian coordinate system. The one-dimensional expansion was defined between constant limits and therefore an implicit assumption of the tensor extension was that the coordinates in the two-dimensional region were bounded between constant limits. However this is not the case in the standard triangular region as the bounds of the Cartesian coordinates (ξ_1, ξ_2) are dependent upon each other.